The Design of Vertical Pressure Vessels Subjected to Applied Forces

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Pressure-vessel codes do not give design methods except for the relatively simple case of cylindrical shells with standard-type heads and openings under uniform pressure. The designer must apply engineering principles when he deals with more complicated structures and loading systems. This paper discusses some design principles that are not covered in the codes. It deals with vessels that are subjected to various applied forces acting in combination with internal or external pressure. The type of vessels considered is limited to cylindrical shells with the longitudinal axis vertical.

NOMENCLATURE

The following nomenclature is used in the paper:

- A = cross-sectional area of shell, sq. in.
- C = acceleration ratio specified by structural codes for use with increased stress values
- D = outside diameter of shell, in.
- e = eccentricity of resultant load, in.
- K = equivalent acceleration ratio for use with basic stress values permitted by vessel codes
- L = length of shell between stiffeners, in.
- M = bending moment due to horizontal loads, in-lb.
- m = numerical ratio depending on R and L
- n = number of lobes into which shell may buckle
- P = end load in addition to external pressure, pound per lineal inch
- p = internal pressure, psi
- R = outside radius of shell, in.
- R = Reynolds number
- r = numerical ratio defined by Equation [1]
- r' = numerical ratio defined by Equation [2]
- r'' = numerical ratio defined by Equation [3]
- S_a = basic allowable stress value permitted by codes
- S_b = bending stress on outermost fiber, psi
- S_c = longitudinal compressive stress in shell, psi
- S_H = stress produced by seismic loads for an acceleration ratio of unity, psi
- S_t = longitudinal tensile stress in shell, psi
- S_v = stress produced by vertical loads, psi
- t = thickness of shell, in.
- W = weight above section under consideration, lb.
- We = collapsing pressure for external pressure acting on sides of vessel only, psi
- We' = collapsing pressure for external pressure acting on sides and ends of vessel, psi
- We'' = collapsing pressure for external pressure on sides and ends when acting in conjunction with an axial compression of P pounds per lineal inch of shell, psi
- W_a'' = We'' divided by a factor of 4 against collapse
- W_a' = We' divided by a factor of 4 against collapse
- Z = section modulus of shell, cu in.

$\alpha = \frac{2P}{W_cR}$ = axial compression per lineal inch due to externally applied loads divided by axial compression set up by that value of external pressure in pounds per square inch which acting by itself would produce collapse.

INTRODUCTION

The pressure-vessel codes (1, 2)[2] give a list of the principal loading conditions that the designer should consider in designing a vessel. These conditions may be divided into pressure loadings and applied forces. Pressures are applied either internally or externally over the surface of the vessel. Applied forces act either at local points or throughout the mass of the vessel.

The codes furnish the designer with a list of approved materials and the maximum stress values in tension permitted over their usable range of temperatures. The design rules in the codes are limited to vessels of cylindrical or spherical shape under internal or external pressure, and to heads and nozzle attachments for such vessels. Rules for more complicated types of construction and for loadings other than that due to pressure are beyond the scope of the code. To include such rules would turn the code into a design handbook. And it would restrict the designer in working out his design in accordance with acceptable engineering principles. The code requires that he "shall provide details of construction that will be as safe as those provided by the rules of the code."

This paper discusses some problems of design of cylindrical pressure vessels that have their axes vertical and are subjected to applied forces in addition to internal or external pressure. The vertical forces considered are the weight of the vessel and its contents and the weight of any attachments to the vessel. The horizontal forces include wind pressures, seismic forces, and piping thrusts.

LOADS

The vertical loads consist primarily of forces due to gravity, that is, to weight. The vertical component of piping thrusts also must be considered. Liquid contents normally are carried by the bottom head and the vessel supports. But in fractionating columns, the weight of the liquid on internal trays is transferred into the shell. Part of the weight of stored solids is transferred into the shell by friction. The weights of attachments that are eccentric to the axis of the vessel produce bending moments which must be considered in the design.

Wind Load. The force per unit area exerted by the wind depends on a number of factors, including wind velocity, height above ground, and drag coefficient. This last includes height-to-diameter ratio and shape factor. ASA Standard A58.1-1945, Minimum Design Loads in Buildings and Other Structures (3), gives a map of the United States showing isograms of equal velocity pressures. It includes also a discussion of methods of arriving at wind pressures from Weather Bureau wind velocities.

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The discussion does not consider the effect of velocity on the drag coefficient. The drag coefficient, or friction factor, for a given shape varies with the Reynolds number. For a circular cylinder, the coefficient is practically constant for values of Reynolds number between **R** = 20,000 and **R =** 200,000. Above **R =** 500,000, the coefficient drops to less than half its value in the lower range (4).

The values of wind pressure used by designers are usually taken from structural codes or from a purchaser's specification. All such values appear to be based on the use of a drag coefficient for Reynolds number in the region between **R =** 20,000 and **R =** 200,000. The value of Reynolds number for a circular cylinder is equal to 9100 *DV,* where *D* is the diameter in feet and *V* is the wind velocity in miles per hour. With a value of *DV* greater than 55, the drag on the vessel would be less than half that specified in the codes. Thus the codes might well consider the advisability of reducing the wind pressures to be wed with circular vessels.

Earthquake. The behavior of a structure in an earthquake is one of vibration under variable conditions of acceleration. For a discussion of the problem from a dynamics approach, see the paper, "Lateral Forces of Earthquake and Wind," by a Joint Committee of the San Francisco, California Section, ASCE, and the Structural Engineers Association of Northern California (5).

The usual simplified approach to the problem is based on the assumption that the structure is a rigid body which undergoes the accelerations of the supporting ground. The horizontal force which acts on the structure is equal to its mass times the ground acceleration, and has the same ratio to the weight as the ground acceleration has to that of gravity. Structural codes give values of this ratio that are based on engineering experience and judgment.

The most widely used code with rules for earthquake design is the Uniform Building Code of the Pacific Coast Building Officials Conference (6). It gives acceleration ratios, or C-factors, for three zones of earthquake intensity. The ratios specified for tanks, smokestacks, standpipes, and similar structures are 0.025, 0.05, 0.10, respectively, for the three zones.

Stress Increase. Structural codes provide for the use of increased allowable stress values when loads due to wind pressure or seismic effect are included. The increase most commonly specified is 33 1/3 per cent. The load values for wind pressures and acceleration ratios in these codes were set with this increase in mind. Pressure-vessel codes do not provide for such stress increases. To maintain consistency between the two types of codes, the acceleration ratios given in structural codes should not be used directly in designing pressure vessels. A correction should be made to offset the effect of the increase in allowable stress values permitted in the structural codes. The correction may be made by modifying the value of the acceleration ratio in such away that the stresses computed in a shell or structure are the same with either type of code.

$$\frac{S_v + KS_H}{S_a} = \frac{S_v + CS_H}{1.33S_a}$$

Clearing of fractions

$$S_v + KS_H = 0.75S_v + 0.75CS_H$$

or

$$K = 0.75C - 0.25\frac{S_v}{S_H}$$

The vessel designer who must meet, the earthquake requirements of a structural code is in somewhat of a dilemma. The structural code specifies certain acceleration ratios but permits him to use a new of 4/3 X 20,000 = 26,670 psi for a vessel constructed of plate to Specification SA-285, Grade D. The pressure vessel codes specify an allowable stress value of 13,750 psi but leave the selection of an acceleration ratio to the designer. Until the vessel codes permit the use of increased stress values for earthquake loads, the equivalent acceleration ratios K derived in the foregoing offers a reasonable solution to the dilemma. Even then, the designer comes up with a thicker shell than the structural rules require because of the lower basic stress values specified in the vessel codes.

It would be possible to calculate a reduced wind load in the same way as was done for earthquake load. There is this difference, however. The specified wind loads are based on measurements of the forces exerted by the wind on structures, and have less of the element of judgment which is involved in setting the earthquake-acceleration ratios. While there is justification for changing the judgment factors for earthquake when the stress values that helped to determine the factors are changed, a parallel change in wind loads which are based on wind intensity has little justification . A direct approach would be for the vessel codes to permit the use of the one-third increase in stress values for wind and earthquake, when designers are to use the wind-load and earthquake-load values that are given in the structural codes.

STRESS DETERMINATION

The vertical loads on the vessel set up compressive stresses in the shell, and also bending stresses when the resultant force does not coincide with the axis of the vessel. The stresses set up at any section of the shell by the vertical loads are given by equations

$$S_c = \frac{W}{A} = \frac{W}{\pi Dt} \qquad \text{and} \qquad S_b = \frac{4We}{\pi D^2 t}$$

The horizontal loads on the vessel produce bending stresses in the shell. The bending moment at any section is equal to the resultant of the horizontal forces above the section multiplied by the distance between the line of action of the resultant and the section. The stress, set up in the outermost fiber of the shell by the action of horizontal loads, is equal to

$$S_b = \frac{4M}{\pi D^2 t}$$

The stresses due to external loads must be considered in combination with those due to pressure in determining the required shell thickness. For internal pressure, the stresses may be combined by simple addition. For external pressure, a more complicated procedure is required.

ALLOWABLE STRESS VALUES

The pressure-vessel codes give tables of allowable stress values in tension for all materials approved for code use. The ASME Subcommittee on Unfired Pressure Vessels has approved for submission to the Main Committee a method for obtaining allowable stress values in compression for ferrous materials. These values are obtained from the charts given in the code for determining the thickness of shells and heads under external pressure. The wording of the proposed method is as follows, except for the addition of paragraph references.

The maximum allowable compressive stress to be used in the design of cylindrical shells, subjected to loadings that produce longitudinal compressive stresses in the shell, shall be the smaller of the following values:

1 The maximum allowable tensile-stress value permitted in Par UG - 23 (a).

2 The value of the factor B determined from the applicable chart in Subsection C for determining the required thickness of

shells and heads under external pressure, using the following definitions for the symbols on the chart (References are to Section VIII of the ASME Boiler and Pressure Vessel Code.):

t_h = minimum required thickness of shell plates, exclusive of corrosion allowance, in.

L_1 = inside radius of cylindrical shell, in.

The value of factor B shall be determined from the applicable chart of Subsection C in the following manner:

Step 1. Assume a value of t_h Determine the ratio $L_1/100t_h$

Step 2. Enter the left-hand side of the chart in Subsection C for the material under consideration at the value $L_1/100t_h$ determined in Step 1.

Step 3. Move horizontally to the line marked "sphere line."

Step 4. From this intersection move vertically to the material line for the design temperature. (For intermediate temperatures, interpolations may be made between the material lines on the chart.)

Step 5. From this intersection move horizontally to the right hand side of the chart and read the value of B. This is the maximum allowable compressive stress value for the value of t_h used in Step 1.

Step 6. Compare this value of B with the computed longitudinal compressive stress in the vessel, using the assumed value of it. If the value of B is smaller than the computed stress, a greater value of t_h must be selected and the procedure repeated until a value of B is obtained which is greater than the computed compressive stress for the loading on the vessel.

The joint efficiency for butt-welded joints may be taken as unity for compressive loading.

DESIGN WITH INTERNAL PRESSURE

The axial stresses set up in the shell may be classified under three types: (a) The longitudinal stress produced by the internal pressure; (b) the uniform compressive stress produced by the sum of the weights assumed to act along the axis of the vessel; (c) the bending stress produced by the horizontal loads and by the resultant weight when eccentric to the axis of the vessel.

Tests carried on at the University of Illinois (7, 8) indicate that a somewhat higher computed stress is required to produce failure under combined bending and compression than under compression alone. Thus we may safely combine compressive stresses due to bending with those due to uniform compression, and design the vessel shell as though these stresses were all due to uniform compression.

The tension side of the shell has its highest stress when the vessel is under pressure. On the compression side, the highest stress occurs when the internal pressure is not acting. The stresses set up in the shell for these two conditions are

Tension $$S_t = \frac{pD}{4t} - \frac{W}{\pi Dt} + \frac{4We}{\pi D^2 t} + \frac{4M}{\pi D^2 t}$$

Compression $$\cdot S_c = -\frac{W}{\pi Dt} - \frac{4We}{\pi D^2 t} - \frac{4M}{\pi D^2 t}$$

The factor W includes all the vertical loads and the factor *M* includes all the moments due to horizontal loads for the loading condition under consideration. A value of shell thickness must be selected so that these stresses are not greater than the allowable stress values, taking into account the applicable joint efficiencies.

DESIGN WITH EXTERNAL

The code charts for determining the required thickness of shells under external pressure have been developed for the condition of a uniform pressure on the cylindrical surface and the heads of the vessel. The longitudinal compressive stresses set up in the shell by weight and lateral forces have an effect similar to that produced by subjecting the heads of the vessel to a higher external pressure than that which acts on the shell.. The charts can be used to give an approximate solution for such a load condition by a suitable change in the vertical scale on which the factor B is read. The nature of the approximations will be discussed later. In practice, it is simpler to leave the scale unchanged and make the adjustment in the value of pressure to be used in reading the chart.

In reference (9) Sturm gives a method of dealing with end loads on the heads. This method can be used as the basis for the design of a vessel under external pressure and subjected to applied loads.

Using Sturm's Equation [45], the ratio of the collapsing pressure W_e and W_e' is equal to

$$r = \frac{W_e'}{W_e} = \frac{F}{F = \frac{\pi^2 R^2}{2L^2}}$$

or $$r = \frac{n^2 - 1}{n^2 - 1 + m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots[1]$$

where $$m = \frac{\pi^2 R^2}{2L^2} = \frac{\pi^2 D^2}{8L^2} = \frac{1.23D^2}{L^2}$$

and F is given[3] as a approximately equal to $n^2 - 1$, where n is the number of lobes into which the shell may buckle. By Sturm's Equation [46], the ratio of collapsing pressures W_e and W_e'' is equal to

$$r' = \frac{W_e''}{W_e} = \frac{F}{F + \frac{\pi^2 R^2}{2L^2} + \frac{2P}{W_e' R}\frac{\pi^2 R^2}{2L^2}}$$

or $$r' = \frac{n^2 - 1}{n2 - 1 + m + m\alpha} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots[2]$$

In this equation $\alpha = 2P/(W_e'R)$, where P is the axial compression per lineal inch due to the externally applied loads, and $W_{e'}R/2$ may be looked on as the axial compression per lineal inch in the shell, if the collapsing value of the external pressure were acting on the ends of the vessel.

Since the ASME Code charts are made up on the basis of the collapsing pressure $W_{e'}$, the ratio of $W_{e'}$ to $W_{e''}$ is needed to make use of the charts. This is found by dividing Equation [1] by Equation [2], whence

$$r'' = \frac{W_e'}{W_e''} = \frac{n^2 - 1 + m + m\alpha}{n^2 - 1 + m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots[3]$$

Windenburg and Trilling (10) have developed a chart which gives n as a function of t/D and L/D for pressure on the sides and ends of the vessel. This chart is reproduced in Fig. 1. A comparison of Strurm's Figs. 4 and 8 indicates that the values of n for different values of t/D and L/D change very little between the condition of pressure on the sides only and that of pressure on both sides and ends. Thus Fig. 1 should give satisfactory values of n for external loading conditions for which α is not much greater than one. For large values of α, the shell will buckle in fewer lobes than the number given in Fig. 1, or it may even fail by plastic flow without the formation of any lobes. For values of α greater than one, the vessel should also be checked as a cantilever beam, including the axial stress due to external pressure in the computations.

The relation between W_e' and $W_{e''}$ given by r'' is a ratio. Hence

Reference (9), p.25.

the value of r″ can be used equally well for the relation between allowable external working pressures. Thus the code charts can be used to determine the required thickness with external loads and moments by using an equivalent design external pressure $W_{a'}$ equal to

$$W_a' = \frac{n^2 - 1 + m + m\alpha}{n^2 - 1 + m} \times W_a'' \qquad [4]$$

where W_a' and W_a'' are equal to the values of W_e' and W_c'' divided by the factor of 4 against collapse.

The use of Equation [4] in connection with the code charts implies that the pressure on the sides of the vessel is increased in the same ratio that the applied vertical forces increase the axial compression in the shell. Since the applied loads do not increase the circumferential compression in the shell, the use of Equation [4] gives answers that are somewhat on the side of safety. The design procedure will be illustrated by an example.

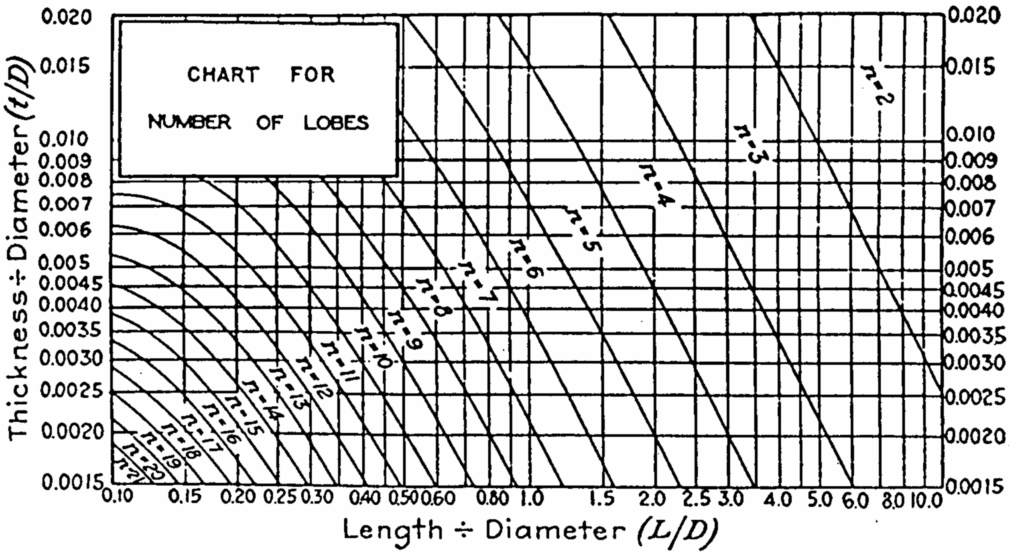

Fig. 1 Number of Lobes n Into Which a Shell Will Collapse When Subjected to Uniform Collapsing Pressure on Sides and Ends

Example. Given a cylindrical vessel fabricated from SA-285, Grade B material, to operate under an external pressure of 15 psi (vacuum) at 200 F. The vessel is 10 ft diam and 100 ft high with stiffening rings spaced 6 ft apart. The total vertical load is 200,000 lb and the moment of the external forces at the bottom head seam is 2,000,000 ft-lb. What is the required shell thickness?

Solution. The maximum compressive load in pounds per lineal inch due to weight and moment is

$$P = \frac{W}{\pi D} + \frac{4M}{\pi D^2}$$

$$= \frac{200{,}000}{120\pi} + \frac{8{,}000{,}000 X 12}{14{,}400\pi}$$

$$= 530 + 2120 = 2650 \text{ lb. per lineal in. of circumference}$$

$$\alpha = \frac{2P}{W_e' R} = \frac{4P}{W_e' D} = \frac{P}{W_a' D} = \frac{2650}{15 X 120} = 1.47$$

Assume t=3/8 in.; then D/t = 320, t/D = 0.00312, L/D = 0.6, m =1.23/0.36 = 3.42, and from Fig. 1, n = 9

$$W_a' = \frac{81 - 1 + 3.42 + 3.42 X 1.47}{81 - 1 + 3.42} X 15 = 15.9\, psi$$

Enter Fig. UCS-28 in the ASME Code for Unfired Pressure Vessels, Section VIII, 1952 edition, with L/D = 6300 and D/t = 320. Then B = 6300 and W_a' = 6300/320 = 19.7 psi.

Since a is greater than 1, check the vessel as a cantilever beam. The axial stress due to vacuum is equal to

$$\frac{15 X 120}{4} = 450 \text{ lb. per lineal in.}$$

This gives a total axial stress of

$$450 + 2650 = 3100 \text{ lb. per in. or } 3100 / 0.375 = 860 \text{ psi}$$

$$L_1 / 100 t_h = \frac{60}{100 X 0.0375} = 1.6$$

Enter Fig. UCS-28 with $L_l/100t_h$ = 1.6 and read B = 10,300. Thus the assumed thickness of 3/8 in. is satisfactory.

SUMMARY

External loads applied to vertical pressure vessels produce axial loading and bending moments on the vessel. These result in axial tensions and compressions in the shell, which must be combined with the effects of the pressure loading to give the total longitudinal stress acting in the shell.

The design method to be used depends on whether the longitudinal stress in the shell is tension or compression, and on whether the vessel is subjected to internal or external pressure.

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DISCUSSION

M. B. HIGGINS.[4] The author has presented a very satisfactory procedure for the design of vertical cylindrical shells that may be a portion of a pressure vessel or its supporting skirt. He has brought together structural practices, research work, and theoretical considerations of the problem. They have been combined in a manner that permits the use of the existing precepts and charts in the ASME Unfired Pressure Vessel Code for such design. This paper should lead to the adoption of suitable rules covering the design of vertical cylindrical shells for inclusion in the Code.

W. SAMANS.[5] Does the author have any evidence of failure of pressure vessels resulting from earthquakes? Also, is there any record of the number of earthquakes in which pressure vessels have come through without damage.

D. J. BERGMAN.[6] The writer would like to comment further in regard to the author's statements in connection with the earthquake in 1935. At that time a refinery, in which was installed a vessel 8 ft diam X 86 ft long on a concrete foundation, was shaken up pretty badly. Anchor bolts on opposite sides of the tower along the principal axis of movement were stretched more than 2 in. The vessel was in service but no damage was sustained by it or by the piping.

It would be an excellent idea for the author to add a statement on the effect of vibrations which are set up by winds at given speed which will set up vibrations coinciding with the natural vibration period of the structure.

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AUTHOR'S CLOSURE:

There is the case in 1952 of a refinery being hit by an earthquake at the epicenter. All the vessels came through without damage. There was some stretching of the anchor bolts. As a matter of fact, the rules of the Uniform Building Code of the Pacific Coast were confirmed as a result of the earthquakes in 1952. Structures of all kinds which had been designed under the Code rules and where the detailing was done under competent engineers came through satisfactorily. Those structures constructed under substandard methods did not come through the earthquake. The author examined a water tank which had been raised from its original height. The tank was mounted on six concrete columns. In doubling the height of the columns, the anchor bolts were the only reinforcement provided between the old and the new concrete. That tank was a total loss. Structures that were designed under Code rules came through in good shape. One must expect some cracks in plaster construction but other buildings did not do as well.

We have given some study to stack vibration. It takes place at the natural period of the stack and we have reason to believe that there is a relation between the diameter and the height of the stack, the height being related to the period more or less. If the stack is higher or lower than given by this relation, severe vibration does not occur. We have only one example where a vibration was set up in a stack at ground level. This stack was set on a concrete foundation. The anchor bolts were not pulled up tight enough. After further tightening, the vibration stopped.