Stresses in a Pressure Vessel With a Conical Head [1]

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This paper presents the results of computations for determining the stresses in a pressure vessel with a conical head. The accurate bending theory of shells is used to evaluate the local bending stresses in the neighborhood of the junction of the conical head and the cylindrical body. Tables show the magnitudes of the shear stress, the circumferential stress, and the axial stress at the junction as multiples of *pd/2t.* For the axial and circumferential stresses, the tables show the magnitude and sense of the stress on both the internal and external surfaces of the vessel. Additional results show the magnitude and location of the maximum stress (of each of the three types) in the cylinder. Curves are given showing the maximum stresses for values of cone apex angle, ratio of conical head thickness to cylinder thickness, and ratio of cylinder diameter to cylinder thickness which will include most of the vessels encountered in practice.

Tables of influence numbers for the conical head are presented. These can be utilized in many problems which require attaching a conical shell to some other elastic structure. A discussion of the mathematical procedure is contained in an Appendix

NOMENCLATURE

The following nomenclature is used in the paper:

δ = radial deflection of middle surface at junction, positive outward, in.
δ' = rotation of middle surface at junction, positive as indicated in Fig. 1
T = thickness of cone, in.
t = thickness of cylinder, in.
E = modulus of elasticity, psi
p = internal pressure, psi
μ = Poisson ratio = 0.3 for steel
α = half vertex angle of cone
ξ = cone parameter = $2m\sqrt{(y/T)}\cot\alpha$
ξ_o = cone parameter at base = $m\sqrt{(d/T)\cot\alpha\csc\alpha}$
d = diameter of cone base, diameter of cylinder, in.
m^4 = $12(1-\mu^2)$
β = $m/\sqrt{dt}$
λ = $\beta M_o/Q_o$
y = distance of point from apex of cone measured along middle surface, in.
Q_y = shear force per unit length, lb per in.
M_y, M_θ = bending moments per unit length, lb
N_y, N_θ = tensile forces per unit length, lb per in.
X_1 = location of maximum shear stress in cylinder
X_2 = location of maximum axial stress in cylinder
X_3 = location of maximum circumferential stress in cylinder
I = stress index = $\dfrac{stress}{pd/2t}$
$\left.\begin{array}{l} a_1, a_2, \ldots a_6 \\ b_1, b_2, \ldots b_6 \end{array}\right\}$ = influence numbers

INTRODUCTION

This paper, dealing with the stresses in a pressure vessel with a conical head, is part of a continuing program instituted by the Design Division of the Pressure Vessel Research Committee of the Welding Research Council of the Engineering Foundation. This program, which consists of both analytic and experimental investigations, is intended to benefit engineers engaged in the design and manufacture of pressure vessels.

The paper is the first of a series which ultimately will cover the kinds of vessel heads in common use. Two additional papers dealing with flat and hemispherical heads will be published later. Each design paper consists of tables and curves for determining the maximum stress in a pressure vessel under the specified end closure.

The elements of the theory (which are well known)[4] are not repeated here and the report limits itself to a description of the computational procedure together with a discussion of the results. The problem logically consists of two parts: (l) The shear force Q_0 and axial bending moment M_o at the cone-cylinder junction (see Fig. 1), must be determined from the continuity of the radial displacement and rotation at the junction. (2) When these are known we may determine the shear, circumferential and axial stresses at the junction in the cylinder and in the cone. The maximum stresses in the cylinder other than at the junction also can be found. Details are given in the Appendix. Each stress is divided by *pd/2t,* the circumferential stress in a thin unrestrained cylinder under uniform pressure, to form the stress index denoted by *I*. The values at the junction are distinguished from the cylinder maxima by subscripts *j* and *m*, respectively. The subscripts *s*, *a*, *c*, denote sheer, axial, and circumferential, respectively. There are thus nine stress indexes, denoted by I_{sm}, I_{am}, and I_{cm} and I_{sj}, I_{aj}, I_{cj} It will be clear from the context whether the last three of these refer to the cylinder or the cone. The stress index in the cone I_{aj}, will be called axial though properly it has the direction of *y*, Fig. 1. The stresses in the cone other than at the junction have not been computed.

The stress ratio *I* has two distinct advantages. It reduces the magnitude of the results and enables one to tell immediately

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[1] This work is part of a continuing program instituted in 1946 by the Design Division of the Pressure Vessel Research Committee of the Welding Research Council of the Engineering Foundation New York, N. Y.

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[4] The theory of shells is discussed in "Theory of Plates and Shells," by S. Timoshenko, first edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1940, Chapt. XII A general mathematical treatment of pressure vessels will be found in "The Basic Elastic Theory of Vessel Heads Under Internal Pressure," by G. W. Watts and R. Burrows, Trans. ASME, Vol. 71, 1949, pp. 55-73.

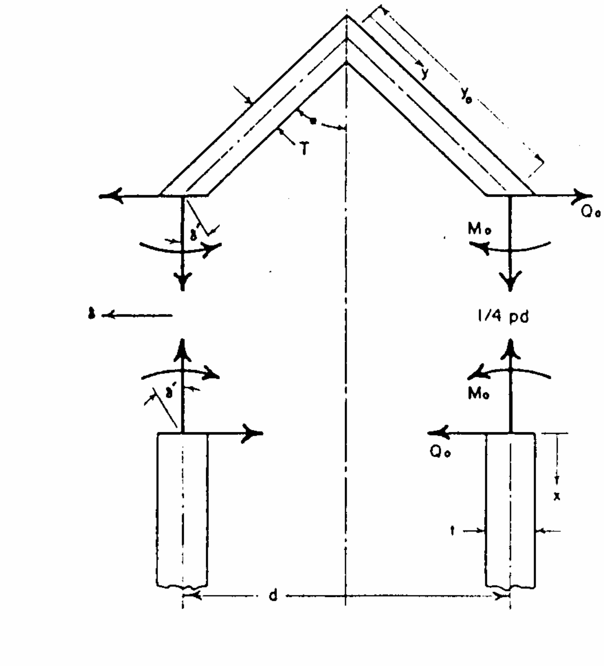

Fig. 1 LOADS AT JUNCTION OF HEAD AND SHELL

whether a stress is greater or less than the hoop stress in a thin cylinder. It frequently is substantially linear when plotted against the diameter-thickness ratio, d/t, of the cylinder. By contrast, a curve of stress divided by pressure is more nearly parabolic.

Since the theory of the shell is concerned with middle surface displacements, conditions at the junction of a physical shell do not correspond precisely with the junction conditions formulated here. This is indicated in Fig. 2 where it is apparent that

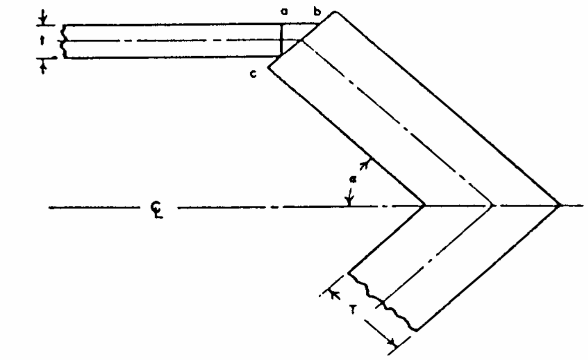

FIG. 2 DETAIL AT JUNCTION OF HEAD AND SHELL

the cylinder and head overlap. To state this differently, we have a cylinder, cone, and a transition element of triangular cross section (*a b c*). Some discrepancy, even in the absence of yielding, is to be expected between the computed results and any results obtained experimentally because of the effect of this transition element. Questions of this sort constitute a problem in local stresses and lie outside the scope of shell theory.

The determination of a correction for this local state of stress is being investigated by a PVRC Design Division Subcommittee. Their findings, which will be published, can be used to extend the results tabulated in the present paper.

ANALYSIS

The deflection and rotation at the junction can be expressed in the forms

$$\frac{Et\delta}{pd^2}=\frac{M_o}{pd^2}a_1+\frac{Q_o}{pd}a_2+a_3=\frac{M_o}{pd^2}a_4+\frac{Q_o}{pd}a_5+a_6 \ldots [1]$$

$$\underbrace{\frac{ET^2\delta'}{m^2pd^2}=\frac{M_o}{pd^2}b_1+\frac{Q_o}{pd}b_2+b_3}_{\text{(for cone)}}=\underbrace{\frac{M_o}{pd^2}b_4+\frac{Q_o}{pd}b_5+b_6}_{\text{(for cylinder)}} \ldots [2]$$

The quantities a_1, a_2, a_3, b_1, b_2, and b_3, are dimensionless "influence numbers" for the conical head. Thus a_3 represents the effect of unit pressure in producing a deflection $ET\delta/d^2$ at the base of the cone. Similarly, a_4, a_5, a_6, b_4, b_5, *and* b_6 are dimensionless influence numbers for the cylinder. The definition of all influence numbers is given in the Appendix.

The dimensionless ratios M_o/pd^2 and Q_o/pd computed from Equations [1] and [2], are sufficient to determine the stress indexes listed in Table 1.

RANGE OF CALCULATIONS

The values of T/t used are

For α = 0 deg T/t = 0.8, 1, 1.25
For α = 15 deg T/t = 0.8, 1, sec 15° (1.0353)
For α = 30 deg T/t = 0.8, 1, sec 30° (1.1547)
For α = 45 deg T/t = 0.8, 1, 1.2, sec 45° (1.4142)
For α = 60 deg T/t = 0.8, 1, 1.2, 1.6, sec 60° (2.0)

For each half cone angle, values of ξ_o are selected which will lead to seven to nine values of d/T extending from 3 to 500. The values of d/T are not evenly spaced but fall mostly between 5 and 100, which is the range occurring frequently in practice. Non-integer initial values of ξ_o were selected so that d/T = 4, 10, 40, 80, 100, 300, 500 at α = 45 deg. A few additional values of ξ_o are required at other angles. The 16 values of ξ_o chosen yield 121 values of d/t in the range 3 to 500. Table 2 shows the ξ_o chosen and the values of T/t and α for which each ξ_o is used.

The numerical results are sufficiently complete to allow the designer to interpolate for most values of α, T/t, and d/t encountered in practical problems.

DISCUSSION OF RESULTS

(a) Two Cylinders. If the thickness *T* refers to a cylinder which replaces the conical head in Fig. 1, the stress indexes for T/t = 0.8 are shown in Table 8.[5] Only the shear stress varies with change in diameter-thickness ratio, d/T or d/t. The maximum circumferential stress in the thinner cylinder is the largest stress. It is 26 per cent greater than the hoop stress $pd/2t$. For equal thicknesses the moment M_o, and a shear force Q_o, vanish and the circumferential index is 1 everywhere, while the axial index is 1/2 everywhere. There is no shear stress.

We should expect the stresses to be greater in the thinner cylinder. The only exception to this conclusion is the maximum shear-stress index. Since stresses in a cylinder decay exponentially with distance from the junction, the greater shear-stress index in the thicker cylinder results from its closer proximity to the junction.

All stress indexes are either unchanged or decrease with increase in d/t. This corresponds to an increase in the common diameter of the cylinders while the thicknesses remain fixed. Such an increase leads to larger values of the influence coefficients and a corresponding decrease in M_o/pd^2 and Q_o/pd.

That the signs of $Mo/pd2$ and Qo/pd are negative can be seen physically. Under uniform pressure, the thinner cylinder expands more. A negative shear force is necessary to enforce continuity of deflection. The angle turned through by a line element at the junction when unit negative shear force $Qo/pd = 1$.

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[5] Interchanging cylinders, the results also apply when T/t = 1.25.

acts is represented by b_2 in the left cylinder and b_5 in the right. But b_2 = —1/2 and b_5 = —(1/2) $(T/t)^2$ = - 0.32. This means that the left cylinder rotates in a positive sense by an amount 0.18(Q_o/pd) more than the right cylinder rotates. A negative bending moment is required to reconcile this difference and maintain continuity of deflection.

TABLE 1
STRESS INDEXES IN CONICAL HEAD AT JUNCTION

$$I_{sj} = 3\frac{t}{T}\left|\frac{Q_y}{pd}\right|_o = 3\frac{t}{T}\left|\frac{Q_0}{pd}\cos\alpha - \frac{1}{4}\sin\alpha\right|$$

$$I_{aj} = 2\frac{t}{T}\left|\frac{N_y}{pd}\right|_o + 12\frac{d}{T}\frac{t}{T}\left|\frac{M_y}{pd^2}\right|_o$$

$$= 2\frac{t}{T}\left|\frac{1}{4}\cos\alpha + \frac{Q_o}{pd}\sin\alpha\right| + 12\frac{d}{T}\frac{t}{T}\left|\frac{M_o}{pd^2}\right|$$

$$I_{cj} = 2\frac{t}{T}\left|\frac{N\theta}{pd}\right|_o + 12\frac{d}{T}\frac{t}{T}\left|\frac{M\theta}{pd^2}\right|_o$$

$$= 2\frac{t}{T}\left\{m^2\sin\alpha\left|\frac{M_o}{pd^2}\left(b_1 + \frac{3\mu\xi_o^2\tan\alpha}{m^4}\right) + \frac{Q_o}{pd}b_2 = b_3\right|\right.$$

$$\left. + \left|\frac{M_o}{pd^2}2a_1 + \frac{Q_o}{pd}\left(2a_2 + \mu\sin\alpha\right) + 2a_3 + \frac{\mu\cos\alpha}{4}\right|\right\}$$

STRESS INDEXES IN CYLINDER AT JUNCTION

$$I_{sj} = 3\left|\frac{Q_o}{pd}\right|$$

$$I_{aj} = \frac{1}{2} + 12\frac{d}{t}\left|\frac{M_o}{pd_2}\right|$$

$$I_{cj} = \left|1 - 2m\sqrt{\frac{d}{t}}\frac{Q_o}{pd}(1+\lambda)\right| + 12\mu\frac{d}{t}\left|\frac{M_o}{pd^2}\right|$$

MAXIMUM STRESS INDEXES IN CYLINDER

$$I_{sm} = \frac{I_{sj}\sqrt{1+2\lambda+2\lambda^2}}{e^{\tan-1\frac{1+\lambda}{\lambda}}}$$

$$I_{am} = \frac{1}{2} + 3.4126\sqrt{\frac{d}{t}}I_{sm}$$

$$\frac{x_1}{d} = \frac{\tan^{-1}\frac{1+\lambda}{\lambda}}{m\sqrt{\frac{d}{t}}}$$

$$\frac{x_2}{d} = \frac{x_1}{d} - \frac{\pi}{4}\frac{1}{m\sqrt{\frac{d}{t}}}$$

where

$$\lambda = m\sqrt{\frac{d}{t}}\frac{M_o / pd^2}{Q_o / pd}$$

TABLE: 2 VALUES OF ξ_o

Angle	15°	30°	45°	60°				
$\xi_{0\backslash T/t}$	All	All	All	0.8	1.0	1.2	1.6	2.0
6.1145			x	x	x	x	x	x
9.6678		x	x	x	x	x	x	x
16.0000				x	x	x	x	x
19.3356	x	x	x	x	x	x	x	x
27.3447			x	x	x	x	x	x
30.5723	x	x	x			x	x	x
36.0000				x	x	x	x	
42.0000		x		x	x	x		
52.9528	x	x	x	x				
68.3817	x	x	x					
84.0000		x						
100.0000	x							
107.0000		x						
136.0000	x							
172.0000	x							
218.0000	x							

(b) *Stress Indexes.* 1 All nine stress indexes tend to increase with angle α except for the two circumferential stress indexes I_{cj} at the junction which decrease at sufficiently small values of *d/T* but increase at larger values of *d/T.* This exception is apparently caused by the decrease in a_2 and b_2 which outweighs increases in a_1, b_1, a_3, and b_3 in the cone at the junction. In the cylinder at the junction the shear force reduces the circumferential stress arising from pressure and this reduction is greater than the increase in circumferential stress arising from the bending.

2 *As d/T* increases, the stress indexes I_{cm} (in cylinder), I_{am} (in cylinder), and I_{aj} (in both cone and cylinder) increase. The remaining stress indexes may decrease or increase depending on the values of α and T/t.. These changes are caused primarily by the relative effect of changes in the influence numbers.

3 As *T/t* increases, the stress indexes I_{sj} and I_{aj} in the cone decrease while I_{aj} in the cylinder increases. The index I_{cj} (in the cone) decreases except at α = 15 deg and large *d/T* when it increases, the remaining indexes tend to increase at low *d/T* and decrease at large *d/T* for each value of α. The reasoning here follows the argument given in the preceding discussion.

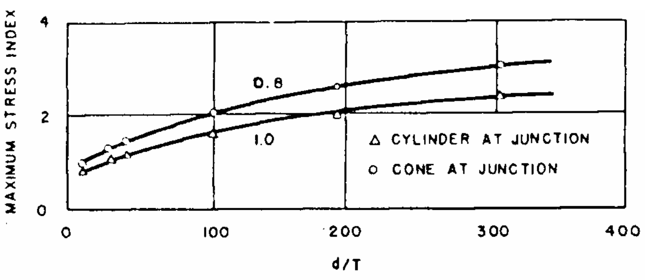
4 The ratios x_1/d, x_2/d, and x_3/d specifying the location of maximum stresses in the cylinder, tend generally to decrease with increasing *d/T,* increasing *T/t*, and decreasing angle α. As the angle decreases we approach more nearly two cylinders and the corresponding decrease in λ leads to maximum cylinder stresses nearer the junction. The increase in *d/T* or *T/t* results in a thicker cone or cylinder. The strain energy induced by the moment and shear force is confined, on the average, to the same volume since M_o/pd^2 and Q_o/pd change relatively slowly. The increase in thickness then requires a decreasing length *x/d* in which the stress is high. Consequently, the maximum stresses should approach the junction.

5 The type of stress index which is the greatest of the nine indexes is shown in Table 3. Generally the axial stress in the cone at the junction is greatest for *T/t* = 0.8, or alternatively for *T/t* = 1.0 and *d/T* sufficiently large. The axial stress in the cylinder at the junction is greatest for *T/t* > 1, or alternatively for *T/t* = 1.0 and and *d/T* sufficiently small. At angles α = 45 deg or less and small *d/T*, the circumferential stress may be greatest. The greatest computed stress index is in the cone at the junction when α = 60 deg and *T/t* = 0.8. The value of this index is 19.825. At α = 15 deg and *T/t* = 1.0, the discontinuity in axial stress at the junction is extremely small. Thus two symbols are inserted in this column of Table 3.

Where the axial stress at the junction does not govern, the values of *d/T* are small and the conical head is not a thin shell as the theory contemplates. Using the legend of Table 3, Figs. 3, 4, 5, and 6 show the variation of maximum stress index with *d/T* where *d/T* extends from approximately 5 to 400.

Figs. 3 to 6 show immediately that it is desirable to make the vertex half angle α and *d/T* as small as possible, that is, the more nearly the cone becomes a cylinder the lower the maximum stress. Also, increasing the thickness of the cone relative to its diameter decreases the maximum stress. The curves suggest that the most favorable thickness ratio is *T/t* = 1.0. When *T/t* < 1.0, the cone is thinner than the cylinder and there is a large axial stress at the junction in the cone. When *T/t* > 1.0, the cone is thicker than the cylinder and there is a large axial stress at the junction in the cylinder. Where it is not possible to have the thickness the same in both cone and cylinder, it is more desirable to make the cone thicker. This is indicated by the fact that curve *T/t* = 1.2, Figs. 5 and 6, leads to lower stress than the curve *T/t* = 0.8. A slight increase in cone thickness at small values of angle α affects this maximum stress very little.

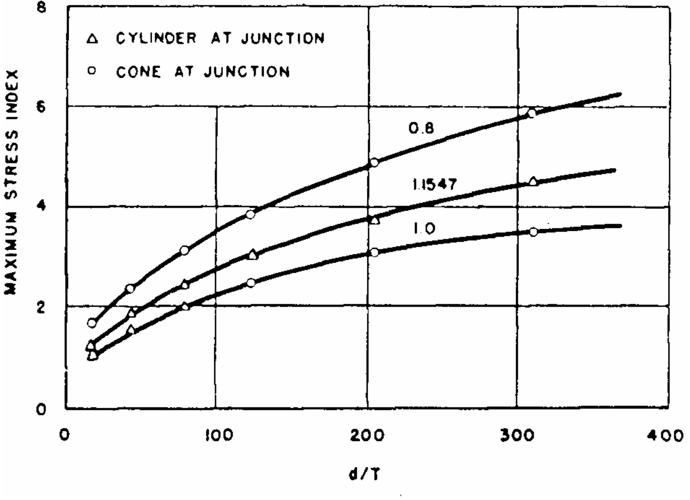
For sufficiently large angle α, variations in *T/t* not too much in excess of 1.0 produce very small changes in maximum stress.



Axial Stress index = axial stress ÷ $\frac{pd}{2t}$;

$\frac{d}{T}$ = diameter of cylinder ÷ thickness of cone

FIG .3 AXIAL - STRESS INDEX VERSUS *d/T* FOR CONICAL HEAD
(α = 15 deg; *T/t* = 0.8 and 1.0.)



Axial Stress index = axial stress ÷ $\frac{pd}{2t}$;

$\frac{d}{T}$ = diameter of cylinder ÷ thickness of cone

FIG. 4 AXIAL - STRESS INDEX VERSUS *d/T* FOR CONICAL HEAD
(α = 30 deg; *T/t* = 0.8, 1.0, 1.1547 = sec 30 deg.)

In conclusion, it is well to note that the possibility of selecting a cone which yields no discontinuities at the junction requires that (1) $a_3 = a_6$ and (2) $b_3 = 0$. The quantities a_3 and a_6 are of opposite sign so that this possibility is precluded.

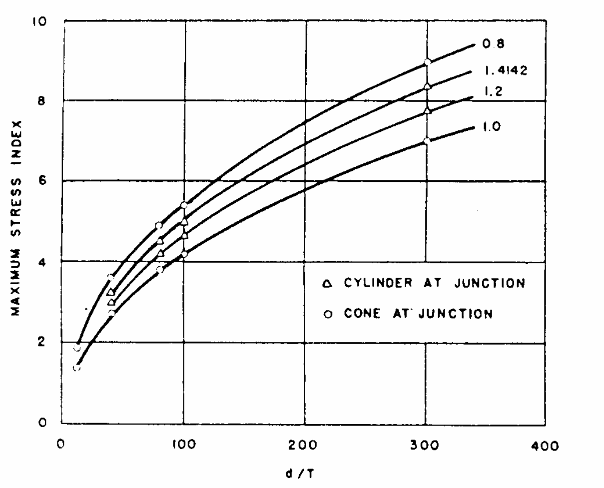
CONCLUSIONS

The general conclusions of interest to designers and engineers engaged in shell problems are summarized as follows:

1 Under uniform pressure *p* alone and no edge constraints but the axial tension (1/4) *pd*, required for equilibrium, a cone deflects inwardly (toward its axis) at its base circle.

2 It is impossible to design a conical head to eliminate moment and shear at the junction since the cylinder always deflects outward and the head inward when an internal pressure is applied to each.

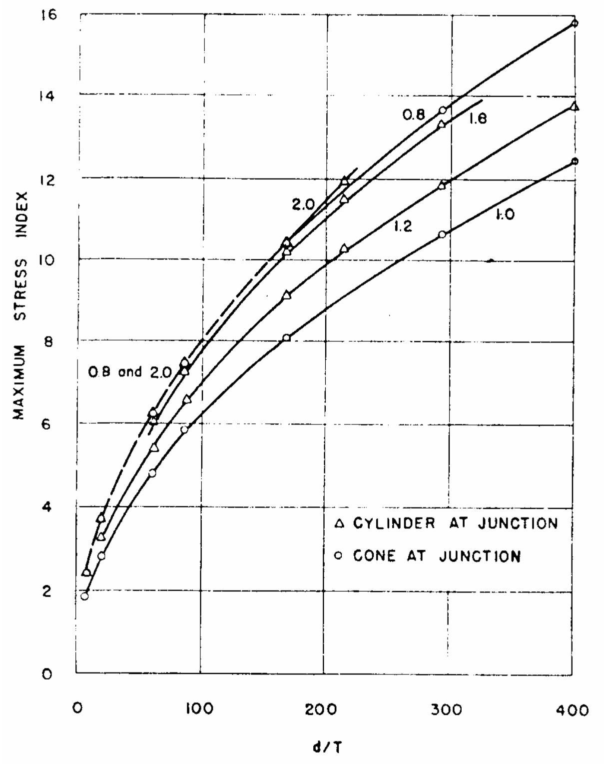
3 In a cylindrical pressure vessel with a conical head, the greatest stress is the axial stress at the junction. It may be located in the cylinder or the cone. To minimize the stress, it is desirable to make the cone and cylinder equally thick if the angle α does not exceed 45 deg. Where this is not possible, the cone should possess the greatest thickness. Angles greater than 45 deg require a thicker head. Reducing the cone vertex angle or the diameter - thickness ratio of the vessels also reduces the stress.



Axial Stress index = axial stress ÷ $\frac{pd}{2t}$;

$\frac{d}{T}$ = diameter of cylinder ÷ thickness of cone

FIG. 5 AXIAL - STRESS INDEX VERSUS *d/T* FOR CONICAL HEAD
(α = 45 deg; *T/t* = 0.8, 1.0, 1.2, 1.4142 = sec 45 deg.)



Axial Stress index = axial stress ÷ $\frac{pd}{2t}$;

$\frac{d}{T}$ = diameter of cylinder ÷ thickness of cone

FIG. 6 AXIAL - STRESS INDEX VERSUS *d/T* FOR CONICAL HEAD
(α = 60 deg; *T/t* = 0.8, 1.0, 1.2, 1.6, 2.0 = sec 60 deg.)

4 The usual theory of shells shows that the maximum axial stress may reach values of 10 or more times the circumferential stress *pd/2t* in the unrestrained thin cylinder. The greatest computed stress

TABLE 3 MAXIMUM STRESS INDEXES FOR CONICAL HEADS

Angle α	15°				30°			45°				60°			
ξo\T/t	0.8	1.0	1.0353 (sec 15°)	0.8	1.0	1.1547 (sec 30°)	0.8	1.0	1.2	1.4142 (sec 45°)	0.8	1.0	1.2	1.6	2.0
6.1145							0	—	—	—0	0	—	—	—	
9.6678				—	—	—	0	—	—	—0	0	—	—	—	
16.0000										0	0	—	—	—	
19.3356	+	☐	☐	0	—	—	0	0	—	—0	0	—	—	—	
27.3447							0	0	—	—0	0	—	—	—	
30.5723	+	☐	☐	0	—	—	0	0	—	—		—	—	—	
36.0000										0	0	—	—		
42.0000				0	—	—				0	0	—			
52.9528	0	—0	—	0	0	—	0	0	—	—0					
68.3817	0	—0	—	0	0	—	0	0	—	—					
84.0000				0	0	—									
100.0000	0	—0	—												
107.0000				0	0	—									
136.0000	0	—0	—												
172.0000	0	—0	—												
218.0000	0	—0	—												

LEGEND:

— Axial - stress index in cylinder at junction	0 Axial - stress index at cone at junction
+ Circumferential - stress index in cone at junction	☐ Maximum circumferential - stress index in cylinder

TABLE 4 INFLUENCE COEFFICIENTS FOR CONE, α = 15 DEG

d/T	a_1	a_2	a_3	b_1	b_2	b_3
3.9231	-6.3685	+1.6824	+0.1020	+3.6189	-0.4913	+0.0199
9.8076	-16.0642	+2.7141	+0.0360	+5.7543	-0.4957	+0.0276
29.4229	-48.4194	+4.7646	-0.0999	+9.9995	-0.4980	+0.0314
49.0382	-80.7915	+6.1753	-0.1941	+12.9205	-0.4986	+0.0322
104.9320	-173.0657	+9.0713	-0.3879	+18.9165	-0.4991	+0.0329
194.0819	-320.2775	+12.3660	-0.6085	+25.7384	-0.4994	+0.0331
310.4304	-512.4285	+15.6604	-0.8291	+32.5598	-0.4995	+0.0333
498.6781	-823.3524	+19.8700	-1.1111	+41.2758	-0.4996	+0.0333

TABLE 6 INFLUENCE COEFFICIENTS FOR CONE, α = 45 DEG

d/T	a_1	a_2	a_3	b_1	b_2	b_3
4.0000	-5.7796	+1.2340	-0.0725	+4.0325	-0.4372	+0.0741
10.0000	-15.5974	+2.1524	-0.2654	+6.6074	-0.4720	+0.0977
40.0000	-64.9345	+4.5965	-0.8558	+13.5059	-0.4913	+0.1163
80.0000	-130.8284	+6.6064	-1.3547	+19.1883	-0.4949	+0.1202
100.0000	-163.7929	+7.4152	-1.5562	+21.4754	-0.4957	+0.1211
300.0000	-493.6916	+13.0171	-2.9547	+37.3186	-0.4980	+0.1235
500.0000	-823.7623	+16.8714	-3.9179	+48.2200	-0.4986	+0.1240

TABLE 5 INFLUENCE COEFFICIENTS FOR CONE, α = 30 DEG

d/T	a_1	a_2	a_3	b_1	b_2	b_3
4.0825	-6.3676	+1.5220	+0.0030	+3.8148	-0.4720	+0.0447
16.3299	-26.5094	+3.2502	-0.2297	+7.7977	-0.4913	+0.0634
40.8248	-66.8682	+5.2433	-0.5138	+12.3988	-0.4957	+0.0683
77.0489	-126.6027	+7.2665	-0.8047	+17.0707	-0.4972	+0.0700
122.4743	-201.5488	+9.2045	-1.0840	+21.5459	-0.4980	+0.0707
204.1239	-336.2996	+11.9299	-1.4770	+27.8398	-0.4986	+0.0713
308.1955	-508.0946	+14.6953	-1.8760	+34.2261	-0.4989	+0.0715
500.0751	-824.8978	+18.7620	-2.4629	+43.6180	-0.4992	+0.0718

TABLE 7 INFLUENCE COEFFICIENTS FOR CONE, α = 60 DEG

d/T	a_1	a_2	a_3	b_1	b_2	b_3
8.4853	-12.2604	+1.5114	-0.3207	+6.9846	-0.4372	+0.1486
21.2132	-33.0871	+2.6361	-0.7559	+11.4443	-0.4720	+0.1809
58.1018	-93.7150	+4.6013	-1.5823	+19.2842	-0.4881	+0.2009
84.8528	-137.7469	+5.6295	-2.0229	+23.3930	-0.4913	+0.2052
169.7056	-277.5290	+8.0911	-3.0837	+33.2351	-0.4949	+0.2103
212.1320	-347.4572	+9.0817	-3.5116	+37.1965	-0.4957	+0.2114
294.1406	-482.6728	+10.7465	-4.2314	+43.8550	-0.4966	+0.2126
400.3580	-657.8465	+12.5860	-5.0271	+51.2120	-0.4972	+0.2135
636.3959	-1047.2780	+15.9426	-6.4797	+64.6377	-0.4980	+0.2145

index is 19.825 for a cone having α = 60 deg, T/t = 0.8, d/T = 509.1167. This stress occurs in the cone at the junction. The high values of stress index suggest early yielding at and near the junction.

SAMPLE PROBLEM

To illustrate the use of curves for determining the secondary bending stresses, assume a 6 - ft - diam vessel with a wall thickness 0.3 in. The internal pressure is 100 psi and the conical head has an apex angle of 120 deg. Therefore d/T = 72/0.3 = 240 and α = 60 deg, pd/2t = 12,000 psi. Using Fig. 6, the following table can be constructed:

T/t =	0.8	1.0	1.2	1.6	2.0
d/T =	300	240	200	150	120
$I = \frac{s}{pd/2t} = =$	13.8	9.6	9.8	9.6	8.9
s =	163000	115000	118000	115000	107000

The magnitude of these stresses indicates, of course, that some yielding and stress relief has occurred in the neighborhood of the junction between the body and the head.

CONVENTION USED IN TABLES 8 THROUGH 23

In this paper the problem of determining the magnitude of the maximum stress has been primarily emphasized. A few additional calculations make it possible to indicate not only the stress magnitude but its sense (whether tension or compression) and its location (whether on the internal or external surface of the vessel). In Tables 8 through 23 the following convention is adhered to:

1 For every circumferential or axial - stress index, two figures are listed for each diameter - thickness ratio. The upper figure represents the value of the stress index at the external surface of the vessel. The lower figure represents the value of the stress index at the internal surface of the vessel. A plus sign denotes tension and a minus sign denotes compression.

2 Every shear - stress index refers to a shear stress at the middle surface of the vessel. The shear stress acting on the cylinder at the junction is positive when it has the direction Q_o in Fig. 1. The shear stress acting on the cone at the junction is positive when its component parallel to the axis of the shell is to left in Fig. 1. The maximum shear stress in the cylinder is positive when it has the direction of Q_o in Fig. 1. The shell wall, to which the stress refers, is to the left.

APPENDIX

INFLUENCE NUMBERS FOR THE CYLINDER

The deflection and rotation produced by loads p, Q_o, M_o, are[6]

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[6] See "Theory of Plates and Shells," S. Timoshenko, first edition, McGraw - Hill Book Company, Inc., New York , N. Y., 1940, pp. 393.

TABLE 8* STRESS INDEX IN TWO CYLINDERS, $\alpha = 0$, $T/t = 0.8$

(Upper values refer to stress at external surface; lower values refer to stress at internal surface.
+ = tension: - = compression.)

Left Cylinder , Thickness t

d/t	Junction Stress Index Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Maximum Stress Index Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Location of Maximum Stress Shear x_1/d	Axial x_2/d	Circum. x_3/d
3.2	-0.0419	+0.4831	+1.0963	+0.0106	+0.4354	+0.9929	0.4546	0.2130	0.8495
		+0.5169	+1.1064		+0.5646	+0.9949			
8.0	-0.0265	+0.4831	+1.0963	+0.0067	+0.4354	+0.9929	0.2875	0.1347	0.5373
		+0.5169	+1.1064		+0.5646	0.9949			
32.0	-0.0132	+0.4831	+1.0963	+0.0033	+0.4354	+0.9929	0.1437	0.0674	0.2686
		+0.5169	+1.1064		+0.5646	+0.9949			
64.0	-0.0094	+0.4831	+1.0963	+0.0024	+0.4354	+0.9929	0.1016	0.0476	0.1899
		+0.5169	+1.1064		+0.5646	+0.9949			
80.0	-0.0084	+0.4831	+1.0963	+0.0021	+0.4354	+0.9929	+0.0909	0.0426	0.1699
		+0.5169	+1.1064		+0.5646	+0.9949			
240.0	-0.0048	+0.4831	+1.0963	+0.0012	+0.4354	+0.9929	0.525	0.0246	0.0981
		+0.5169	+1.1964		+0.5646	+0.9949			
400.0	-0.0037	+0.4831	+1.0963	+0.0009	+0.4354	+0.9929	0.0407	0.0191	0.0760
		+0.5169	+1.1064		+0.5646	+0.9949			

Right Cylinder , Thickness t

d/t	Junction Stress Index Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Maximum Stress Index Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Location of Maximum Stress Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.0	+0.0524	+0.3486	+1.0906	-0.0085	+0.6833	+1.2624	0.4674	0.2514	0.5462
		+0.6514	+1.1455		+0.3167	+0.9218			
10.0	+0.0331	+0.3486	+1.0906	-0.0054	+0.6833	+1.2624	0.2956	0.1590	0.3455
		+0.6514	+1.1455		+0.3167	+0.9218			
40.0	+0.0166	+0.3486	+1.0906	-0.0027	+0.6833	+1.2624	0.1478	0.0795	0.1727
		+0.6514	+1.1455		+0.3167	+0.9218			
40.0	+0.0166	+0.3486	+1.0906	-0.0019	+0.6833	+1.2624	0.1045	0.0562	0.1221
		+0.6514	+1.1455		+0.3167	+0.9218			
80.0	+0.0117	+0.3486	+1.0906	-0.0019	+0.6833	+1.2624	0.1045	0.0562	0.1221
		+0.6514	+1.1455		+0.3167	+0.9218			
100.0	+0.0105	+0.3486	+1.0906	-0.0017	+0.6833	-1.2624	0.0935	0.0503	0.1092
		+0.6514	+1.1455		+0.3167	+0.9218			
		+0.6514	+1.1455		+0.3167	+0.9218			
300.0	+0.0060	+0.3486	+1.0906	-0.0010	+0.6833	+1.2624	0.0540	0.0290	0.0631
		+0.6514	+1.1455		+0.3167	+0.9218			
500.0	+0.0047	+0.3486	+1.0906	-0.0008	+0.6833	+1.2624	0.0418	0.0255	0.0489
		+0.6514	+1.1455		-0.3167	-0.9218			

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*Correction to Table 8. For d/t of 240.0 , under Junction Stress Index, I_{cj} of 1.1964 should be 1.1064.

TABLE 10 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, $\alpha = 15$ DEG, $T/t = 1.0$

(Upper values refer to stress at external surface; lower values refer to stress at internal surface.
+ = tension; - = compression.)

d/t	d/T	Cone at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Cylinder at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
3.9231	3.9231	+0.1019	+0.3860	+0.7936	+0.0955	-0.3686	+0.8038
			+0.6308	+0.8925		+0.6314	+0.8826
9.8076	9.8076	+0.1020	+0.2148	+0.7056	+0.0954	-0.2153	+0.7094
			+0.7841	0.8836		+0.7847	+0.8802
29.4228	29.4228	+0.1006	-0.0592	+0.5031	+0.0968	+0.0588	+0.5040
			+1.0585	+0.8401		+1.0588	+0.8393
49.0381	49.0381	+0.1001	-0.2396	-0.3588	+0.0973	+0.2394	+0.3593
			+1.2392	+0.8033		+1.2394	+0.8029
104.9318	104.9318	+0.0994	-0.6032	+0.0596	+0.0980	+0.6030	+0.0597
			+1.6029	+0.7215		+1.6030	+0.7215
194.0819	194.0819	+0.0990	-1.0123	-0.2827	+0.0984	+1.0122	-0.2827
			+2.0122	+0.6245		+2.0122	+0.6246
310.4304	310.4304	+0.0988	-1.4192	-0.6257	+0.0987	+1.4192	-0.5012
			+2.4192	+0.5257		+2.4192	+0.4581
498.6781	498.6781	+0.0986	-1.9379	-1.0644	+0.0989	+1.9379	-0.9321
			+2.9380	+0.3944		+2.9379	+0.2838

d/t	d/T	Maxima in Cylinder Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Location of Maxima in Cylinder Shear x_1/d	Axial x_2/d	Circum. x_3/d
3.9231	3.9231	+-0.0097	+0.5656	+1.0140	0.5564	+0.3383	0.6360
			+0.4344	+0.9958			
9.8076	9.8076	-0.0074	+0.5787	+1.0169	0.3905	0.2526	0.4409
			+0.4213	+0.9950			
29.4228	29.4228	-0.0067	+0.6240	+1.0266	0.2357	0.1560	0.2647
			+0.3760	+0.9921			
49.0381	49.0381	-0.0066	+0.6581	+1.0339	0.1840	0.1223	0.2065
			+0.3419	+0.9899			
104.9318	104.9318	-0.0066	+0.7300	+1.0494	0.1265	0.0843	0.1418
			+0.2700	+0.9852			
194.0819	194.0819	-0.0066	+0.8129	+1.0672	0.0931	0.0621	0.1044
			0.1871	+0.9800			
310.4304	310.4304	-0.0066	+0.8963	+1.0850	0.0737	0.0491	0.0826
			+0.1037	+0.9747			
498.6781	498.6781	-0.0066	+1.0031	+1.1080	0.0581	0.0388	0.0652
			+0.0031	+0.9678			

TABLE 9 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, $\alpha = 15$ DEG, $T/t = 0.8$

(Upper values refer to stress at external surface; lower values refer to stress at internal surface.
+ = tension: - = compression.)

d/t	d/T	Cone at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Cylinder at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
3.1384	3.9231	+0.1675	+0.4480	+0.9089	+0.0622	-0.3918	+0.9584
			+0.7862	+1.0134		+0.6082	+0.8935
7.8461	9.8076	+0.1517	+0.2423	+0.8007	+0.0754	-0.2583	+0.9498
			+0.9976	+1.0259		+0.7417	+0.8048
23.5383	29.4228	+0.1374	-0.1166	+0.5635	+0.0872	-0.0270	+0.8900
			1.3616	+1.0062		+0.9730	+0.6062
39.2305	49.0381	+0.1327	-0.3508	+0.3961	+0.0911	+0.1234	+0.8390
			+1.5975	+0.9813		+1.1234	+0.4649
83.9455	104.9318	+0.1273	-0.8208	+0.0519	+0.0955	+0.4248	0.7264
			+2.0694	+0.9206		+1.4248	0.1715
155.2656	194.0819	+0.1242	-1.3484	-0.3414	+0.0981	+0.7629	+0.5933
			+2.5981	+0.8452		1.7628	-0.1644
248.3443	310.4304	+0.1224	-1.8730	-0.7351	+0.0996	1.0988	-0.5012
			+3.1234	+0.7672		+2.0988	-0.4581
398.9425	498.6781	+0.1209	-2.5409	-1.2384	+0.1008	+1.5265	-0.9321
			+3.7918	+0.6653		+2.5265	-0.2838

d/t	d/T	Maxima in Cylinder Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Location of Maxima in Cylinder Shear I_{sm}	Axial I_{am}	Circum. I_{cm}
3.1384	3.9231	-0.0047	+0.5282	+1.0061	0.6982	0.4522	0.7873
			+0.4718	+0.9982			
7.8461	9.8076	-0.0049	+0.5464	+1.0100	0.4707	0.3165	0.5270
			0.4536	+0.9970			
23.5383	29.4228	-0.0057	+0.5951	+1.0204	0.2691	0.1800	0.3016
			+0.4049	+0.9939			
39.2305	49.0381	-0.0061	+0.6314	+1.0282	0.20634	0.1374	0.2315
			+0.3686	+0.9916			
83.9455	104.9318	-0.0067	+0.7083	+1.0447	0.1391	0.0920	0.1563
			+0.2917	+0.9867			
155.2656	194.0819	-0.0070	+0.7975	+1.0639	0.1013	0.0667	0.1140
			+0.2025	+0.9810			
248.3443	310.4304	-0.0072	+0.8876	+1.0832	0.0797	0.0522	0.0897
			+0.1124	+0.9752			
398.9425	498.6781	-0.0074	+1.0032	+1.1080	0.0626	0.0409	0.0704
			-0.0032	+0.9698			

TABLE 11 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, $\alpha = 15$ DEG, $T/t = 1.0353$

(Upper values refer to stress at external surface; lower values refer to stress at internal surface.
+ = tension; - = compression.)

d/t	d/T	Cone at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Cylinder at Junction Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
4.0614	3.9231	+0.0939	+0.3545	+0.7762	+0.1003	+0.6380	+0.7897
			+0.6120	+0.8759		-0.3620	+0.8725
10.1536	9.8076	+0.0958	+0.2086	+0.6908	+0.0983	+0.7940	+0.6943
			+0.7572	+0.8639		-0.2060	+0.8707
30.4608	29.4228	+0.0960	-0.0530	+0.4931	+0.0981	+1.0743	+0.4878
			+1.0187	+0.8167		+0.0743	+0.8324
50.7680	49.0381	+0.0959	-0.2255	+0.3521	+0.0982	+1.2593	+0.3424
			+1.1913	+0.7779		+0.2593	+0.7979
108.6334	104.9318	+0.0957	-0.5733	+0.0593	+0.0984	+1.6320	+0.0415
			+1.5391	+0.6928		+0.6320	+0.7208
200.9284	194.0819	+0.0956	-0.9647	-0.2758	+0.0985	+2.0516	-0.3022
			+1.9306	+0.5921		+1.0516	+0.6288
321.3812	310.4304	+0.0956	-1.3543	-0.4901	+0.0985	2.4692	-0.6466
			+2.3202	+0.6115		+1.4692	+0.5349
516.2696	498.6781	+0.0955	-1.8505	-1.0412	+0.0986	+3.0010	-1.0867
			+2.8164	+0.3543		+2.0010	+0.4139

d/t	d/T	Maxima in Cylinder Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Location of Maxima in Cylinder Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.0614	3.9231	-0.0103	+0.5711	+1.0153	0.5442	0.3298	0.6224
			+0.4289	+0.9955			
10.1536	9.8076	-0.0077	+0.5837	+1.0180	0.3817	0.2461	0.4312
			+0.4163	+0.9946			
30.4608	29.4228	-0.0068	+0.6285	+1.0276	0.2313	0.1530	0.2598
			+0.3715	+0.9918			
50.7680	49.0381	-0.0067	+0.6623	+1.0348	0.1808	0.1202	0.2030
			+0.3377	0.9896			
108.6334	104.9318	-0.0066	+0.7336	+1.0501	0.1246	0.0831	0.1397
			+0.2664	+0.9851			
200.9284	194.0819	-0.0065	+0.8158	+1.0678	0.0918	0.0614	0.1030
			+0.1842	+0.9798			
321.3812	310.4304	-0.0065	+0.8986	+1.0856	0.0727	0.0486	0.0815
			+0.1014	+0.9745			
516.2696	498.6781	-0.0065	+1.0045	+1.1083	0.0574	0.0384	0.0644
			-0.0045	+0.9677			

TABLE 12 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, $\alpha = 15$ DEG, T/t = 0.8

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
3.2660	4.0875	+0.2919	+0.2325	+0.6763	+0.1634	+0.2588	+0.7027
			+0.9862	+0.9394		+0.7412	+0.8474
13.0639	16.3299	+0.2705	-0.4625	+0.2855	0.1832	-0.1912	+0.3710
			+1.6977	+0.9420		+1.1912	+0.7857
32.6598	40.8248	0.2569	-1.2039	-0.2261	+0.1957	-0.6691	-0.0622
			+2.4495	+0.8828		+1.6691	+0.6392
61.6391	77.0489	+0.2498	-1.9337	-0.7523	+0.2023	-1.1379	-0.4690
			+3.1848	+0.8163		+2.1379	+0.5137
97.9795	122.4743	+0.2457	-2.6237	-1.2699	+0.2060	-1.5805	-0.9499
			+3.8779	+0.6907		+2.5805	+0.2982
163.2991	204.1239	+0.2421	-3.5868	-1.9910	+0.2094	-2.1980	-1.5662
			+4.8441	+0.5518		+3.1980	+0.0524
246.5564	308.1955	+0.2397	-4.5604	-2.7235	+0.2116	-2.8211	-2.1932
			+5.8187	+0.4056		+3.8211	-0.2006
400.0601	500.0751	+0.2375	-5.9857	-3.8015	+0.2136	-3.7347	-3.1166
			+7.2470	+0.1863		+4.7347	-0.5758

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
3.2660	4.0875	-0.0145	+0.5897	+1.0193	0.6405	0.4014	0.7278
			+0.4103	+0.9943			
13.0639	16.3299	-0.0129	+0.6595	+1.0343	0.3508	0.2313	0.3945
			+0.3405	+0.9899			
32.6598	40.8248	-0.0138	+0.7692	+1.0578	0.2220	0.1464	0.2496
			+0.2308	+0.9828			
61.6391	77.0489	-0.0145	+0.8873	+1.0832	0.1607	0.1057	0.1808
			+0.1127	+0.9752			
97.9795	122.4743	-0.0149	+1.0029	+1.1080	0.1269	0.0833	0.1428
			-0.0029	+0.9678			
163.2991	204.1239	-0.0153	+1.1675	+1.1433	0.0978	0.0640	0.1102
			-0.1675	+0.9573			
246.5564	308.1955	-0.0156	+1.3357	+1.1794	0.0794	0.0518	0.0894
			-0.3357	+0.9466			
400.0601	500.0751	-0.0159	+1.5842	+1.2328	0.0621	0.0405	0.0700
			-0.5842	+0.9307			

TABLE 14 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, $\alpha = 15$ DEG, T/t = 1.1547

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
4.7140	4.0825	+0.1713	+0.1631	+0.5256	+0.2046	+0.1388	+0.5523
			+0.7050	+0.7561		-0.8612	+0.7690
18.8562	16.3299	+0.1767	+0.2620	+0.2398	+0.1974	-0.4253	+0.1932
			+1.1259	+0.6609		+1.4253	+0.7483
47.1404	40.8248	+0.1761	+0.7304	-0.1599	+0.1982	-1.0501	-0.2604
			+1.5948	+0.5287		+2.0501	+0.6697
88.9684	77.0489	+0.1756	+1.1959	-0.5795	+0.1989	-1.6711	-0.7286
			+2.0607	+0.3843		+2.6711	+0.5741
141.4212	122.4743	+0.1753	+1.6378	-0.9855	+0.1993	-2.2604	-1.1796
			+2.5028	+0.2418		+3.2604	+0.4768
235.7020	204.1239	+0.1750	+2.2559	-1.5589	+0.1996	-3.0847	-1.8155
			+3.1212	+0.0.382		+4.0847	+0.3355
355.8735	308.1955	+0.1749	+2.8810	-2.1422	+0.1999	-3.9182	-2.4616
			+3.7464	-0.1705		+4.9182	+0.1894
577.4370	500.0751	+0.1747	+3.7979	-3.0013	+0.2001	-5.1411	-3.4125
			+4.6636	-0.4797		+6.1411	+0.0.279

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.7140	4.0825	-0.0183	+0.6355	+1.0291	0.5331	0.3333	0.6224
			+0.3645	+0.9913			
18.8562	16.3299	-0.0135	+0.6998	+1.0429	0.2961	0.1966	0.3324
			+0.3002	0.9872			
47.1404	40.8248	-0.0.128	+0.8006	+1.0645	0.1916	0.1287	0.2146
			+0.1994	+0.9808			
88.9684	77.0489	-0.0127	+0.9078	+1.0875	0.1405	0.0946	0.1572
			+0.0922	0.9739			
141.4212	122.4743	-0.0126	+1.0119	+1.1099	0.1117	0.0754	0.1250
			-0.0119	+0.9673			
235.7020	204.1239	-0.0126	+1.1595	+1.1416	0.0867	0.0585	0.0970
			-0.1595	+0.9578			
355.8735	308.1955	-0.0126	+1.3098	+1.1739	0.0706	0.0477	0.0790
			-0.3098	+0.9482			
577.4370	500.0751	-0.0126	+1.5315	+1.2214	0.0555	0.0375	0.0620
			-0.5315	+0.9341			

TABLE 14 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, $\alpha = 30$ DEG, T/t = 1.0

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
4.0825	4.0825	+0.2111	-0.1922	+0.5826	+0.1893	+0.1961	+0.6128
			+0.8000	+0.8229		+0.8039	+0.7951
16.3299	16.3229	+0.2083	+0.3303	+0.2606	+0.1925	-0.3274	+0.2649
			+1.3246	+0.7640		+1.3274	+0.7614
40.8248	40.8248	+0.2038	+0.8999	-0.1792	+0.1976	-0.8988	+0.1796
			+1.8976	+0.6586		+1.8988	+0.6597
77.0489	77.0489	+0.2014	+1.4644	-0.6385	+0.2005	-1.4642	-0.6400
			+2.4640	+0.5368		+2.4642	+0.5385
122.4743	122.4743	+0.2000	+1.9994	-1.0821	+0.2021	-1.9998	-1.0841
			+3.0000	+0.4143		+2.9998	+0.4459
204.1239	204.1239	+0.1987	+2.7474	-1.7085	+0.2036	-2.7482	-1.7105
			+3.7491	+0.2371		+3.7483	+0.2385
308.1955	308.1955	+0.1979	+3.5034	-2.3454	+0.2045	-3.5046	-2.3473
			+4.5058	+0.0543		+4.5046	+0.0554
500.0751	500.0751	+0.1971	+4.6125	-3.2831	+0.2054	-4.6140	-3.2849
			+5.6155	-0.2174		+5.6140	-0.2165

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.0825	4.0825	-0.0173	+0.6190	+1.0255	0.5678	0.3539	0.6458
			+0.3810	+0.9924			
16.3299	16.3299	-0.0133	+0.6840	+1.0395	0.3162	0.2093	0.3553
			+0.3160	+0.9882			
40.8248	40.8248	-0.0132	+0.7868	+1.0616	0.2034	0.1358	0.2281
			+0.2132	+0.9817			
77.0489	77.0489	-0.0132	+0.8967	+1.0852	0.1486	0.0994	0.1665
			+0.1033	+0.9746			
122.4743	122.4743	-0.0133	+1.0037	+1.1081	0.1179	0.0789	0.1322
			-0.0037	+0.9678			
204.1239	204.1239	-0.0134	+1.1554	+1.1407	0.0913	0.0611	0.1023
			-0.1554	+0.9581			
308.1955	308.1955	-0.0135	+1.3101	+1.1739	0.0743	0.0497	0.0832
			-0.3101	+0.9482			
500.0751	500.0751	-0.0136	+1.5384	+1.2229	0.0583	0.0389	0.0653
			-0.5384	+0.9336			

TABLE 15 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, $\alpha = 45$ DEG, T/t = 0.8

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
3.2000	4.0000	+0.4261	-0.3965	+0.4414	+0.2679	-0.1005	+0.5194
			+1.2240	+0.8969		+0.8995	+0.7591
8.0000	10.0000	+0.4153	-0.8735	+0.1259	+0.2801	+0.3099	+0.2422
			+1.8725	+0.9099		+1.3099	+0.7288
32.0000	40.0000	+0.3843	-2.0472	-0.9766	+0.3152	+1.3793	-0.6895
			+3.5640	+0.8017		+2.3793	+0.4380
64.0000	80.0000	+0.3725	-2.9680	-1.9334	+0.3286	+2.2245	-1.5025
			+4.8927	+0.6445		+3.2245	+0.1322
80.0000	100.0000	+0.3693	-3.3343	-2.3214	+0.3322	+2.5606	-1.8331
			+5.4197	+0.5743		+3.5606	+0.0032
240.0000	300.0000	+0.3575	-5.8435	-5.0224	+0.3455	+4.8603	-4.1421
			+9.0209	+0.0415		+5.8603	-0.9259
400.0000	500.0000	+0.3538	-7.5588	-6.8850	+0.3497	+6.4307	-5.7384
			+11.4773	-0.4079		+7.4307	-1.5800

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
3.2000	4.000	-0.0234	+0.6431	+1.0307	0.6515	0.4099	0.7396
			+0.3569	+0.9909			
8.0000	10.0000	-0.0202	+0.6950	+1.0419	0.4445	0.2917	0.5002
			+0.3050	+0.9875			
32.0000	40.0000	-0.0220	+0.9256	+1.0914	0.2250	0.1487	0.2529
			+0.0744	+0.9728			
64.0000	80.0000	-0.0234	+1.1380	+1.1370	0.1580	0.1040	0.1778
			-0.1380	+0.9592			
80.0000	100.0000	-0.0238	+1.2256	+1.1558	0.1410	0.0927	0.1586
			-0.2256	+0.9536			
240.0000	300.0000	-0.0254	+1.8440	+1.2885	0.0805	0.0526	0.0907
			-0.8440	+0.9141			
400.0000	500.0000	-0.0260	+2.2749	+1.3810	0.0621	0.0405	0.0700
			-1.2749	+0.8865			

TABLE 16* STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, α = 45 DEG, T/t = 1.0

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
4.0000	4.0000	+0.3262	-0.0118	+0.3765	+0.2887	+0.0015	+0.5276
			+0.9911	+0.7712		+1.0015	+0.7270
10.0000	10.0000	+0.3255	-0.4850	+0.1239	+0.2897	+0.4751	+0.3144
			+1.4653	+0.7237		+1.4751	+0.7193
40.0000	40.0000	+0.3101	-1.7497	-0.8130	+0.3114	+1.7500	-0.8225
			+2.7504	+0.5183		+2.7500	+0.5276
80.0000	80.0000	+0.3041	-2.7669	-1.6373	+0.3200	+2.7713	-1.6483
			+3.7757	+0.3061		+3.7713	+0.3144
100.0000	100.0000	+0.3024	-3.1730	-1.9724	+0.3223	+3.1785	-1.9835
			+4.1840	+0.2159		+4.1785	+0.2236
300.0000	300.0000	+0.2965	-5.9640	-4.3096	+0.3308	+5.9735	-4.3194
			+6.9830	-0.4394		+6.9735	-0.4353
500.0000	500.0000	+0.2946	-7.8743	-5.9233	+0.3334	+7.8850	-5.9323
			+8.8957	+0.9526		+8.8850	-0.9013

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.0000	4.0000	-0.0243	+0.6660	+1.0356	0.5910	0.3750	0.6698
			+0.3340	0.9894			
10.0000	10.0000	-0.0201	+0.7166	+1.0465	0.4042	0.2676	0.4541
			+0.2834	+0.9862			
40.0000	40.0000	-0.0201	+0.9338	+1.0931	0.2083	0.1400	0.2332
			+0.0662	+0.9723			
80.0000	80.0000	-0.0206	+1.1302	+1.1353	0.1473	0.0990	0.1649
			-0.1302	+0.9597			
100.0000	100.0000	-0.0208	+1.2106	+1.1526	0.1317	0.0885	0.1474
			-0.2106	+0.9546			
300.0000	300.0000	-0.0216	+1.7757	+1.2739	0.0757	0.0507	0.0848
			-0.7757	+0.9184			
500.0000	500.0000	-0.0219	+2.1679	+1.3581	0.0585	0.0392	0.0656
			-1.1679	+0.8934			

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* Correction to Table 16. Under Cone at Junction, last entry for I_{cj} should be minus.

TABLE 18** STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, α = 45 DEG, T/t = 1.4142

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
5.6569	4.0000	+0.2173	-0.0015	+0.2861	+0.3154	+0.2133	+0.2697
			+0.7118	+0.5944		+1.2133	+0.6977
19.1421	10.0000	+0.2257	-0.2791	-0.4695	+0.2985	+0.7573	-0.0453
			+0.9871	0.1041		+1.7573	+0.7091
56.5685	40.0000	+0.2244	-1.0402	-0.6367	+0.3011	+2.2811	-1.0475
			+1.7409	-0.1206		+3.2811	+0.6212
113.1371	80.0000	+0.2233	-1.6589	-1.2999	+0.3034	+3.5200	-1.9025
			+2.3611	-0.1824		+4.5200	+0.5095
141.4214	100.0000	+0.2230	-1.9065	-1.5704	+0.3040	+4.0157	-2.2485
			+2.6091	-0.3064		+5.0157	+0.4610
424.2641	300.0000	+0.2219	-3.6122	-3.4610	+0.3061	+7.4285	-4.6540
			+4.3163	-1.4892		+8.4285	+0.1031
707.1068	500.0000	+0.2216	-4.7809	-4.7682	+0.3068	+9.7663	-6.3124
			+5.4584	-1.8173		+10.7663	-0.1526

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
5.6569	4.0000	-0.0244	+0.6982	+1.0425	0.5138	0.3321	0.5801
			+0.3018	+0.9873			
19.1421	10.0000	-0.0197	+0.7523	+1.0542	0.3473	0.2324	0.3892
			+0.2477	+0.9839			
56.5685	40.0000	-0.0181	+0.9646	+1.0997	0.1808	0.1234	0.2018
			+0.0354	+0.9703			
113.1371	80.0000	-0.0180	+1.1527	+1.1401	0.1287	0.0880	0.1435
			-0.1527	+0.9583			
141.4214	100.0000	-0.0180	+1.2292	+1.1566	0.1152	0.0789	0.1285
			-0.2292	+0.9534			
424.2641	300.0000	-0.0180	+1.7642	+1.2714	0.0677	0.0457	0.0744
			-0.7642	+0.9192			
707.1068	500.0000	-0.0180	+2.1341	+1.3508	0.0517	0.0355	0.0756
			-1.1341	+0.8955			

**Table 18. For *d/t* of 19.1421, under Cone at Junction, I_{aj} of 0.9871 should be 0.9872.

TABLE 17 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, α = 45 DEG, T/t = 1.2

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
4.8000	4.0000	+0.2630	-0.0066	+0.3263	+0.3037	+0.1056	+0.2105
			+0.8345	+0.6766		+1.1056	+0.7090
12.0000	10.0000	+0.2678	-0.3684	+0.1155	+0.2956	+0.6220	+0.0406
			+1.1899	+0.5852		+1.6220	+0.7138
48.0000	40.0000	+0.2609	-1.3509	-0.7088	+0.3072	+2.0434	-0.9413
			+2.1815	+0.3014		+3.0434	+0.5847
96.0000	80.0000	+0.2579	-2.1464	-1.4413	+0.3123	+3.1917	-1.7830
			+2.9810	+0.0406		+4.1917	+0.4320
120.0000	100.0000	+0.2571	-2.4650	-1.7396	+0.3137	+3.3605	-2.1241
			+3.3002	-0.0675		+4.6505	+0.3663
360.0000	300.0000	+0.2541	-4.6532	-3.8229	+0.3188	+6.8051	-4.4976
			+5.4928	-0.8423		+7.8051	-0.2646
600.0000	500.0000	+0.2532	-6.1520	-5.2625	+0.3203	+8.9644	-6.1350
			+6.9930	-1.4158		+9.9644	-0.4563

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
4.8000	4.0000	-0.0245	+0.6832	+1.0393	0.5487	0.3515	0.6207
			+0.3168	+0.9883			
12.0000	10.0000	-0.0199	+0.7351	+1.0505	0.3735	0.2488	0.4191
			+0.2649	+0.9850			
48.0000	40.0000	-0.0190	+0.9483	+1.0962	0.1939	0.1316	0.2167
			+0.0517	+0.9713			
96.0000	80.0000	-0.0191	+1.1387	+1.1371	0.1377	0.0936	0.1538
			-0.1387	+0.9592			
120.0000	100.0000	-0.0192	+1.2164	+1.1538	0.1232	0.0838	0.1376
			-0.2164	+0.9542			
360.0000	300.0000	-0.0195	+1.7607	+1.2706	0.0711	0.0484	0.0795
			-0.7607	+0.9194			
600.0000	500.0000	-0.0196	+2.1375	+1.3515	0.0551	0.0374	0.0615
			-1.1375	+0.8953			

TABLE 19 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, α = 60 DEG, T/t = 0.8

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}	Shear I_{sj}	Axial I_{aj}	Circum. I_{cj}
6.7882	8.4853	+0.5659	-1.0969	-0.1370	+0.3937	+0.5838	-0.0288
			+2.2901	+0.9028		+1.5838	+0.6791
16.9706	21.2132	+0.5383	-2.4829	-0.9822	+0.4377	+1.4912	-0.6859
			+3.7397	+0.8211		+2.4912	+0.5088
46.4815	58.1018	+0.5056	-4.8992	-2.6773	+0.4853	+3.0596	-2.1172
			+6.2247	+0.6763		+4.0596	+0.0186
67.8822	84.8528	+0.4996	-6.1454	-3.5936	+0.4997	+3.8638	-2.8952
			+7.4916	+0.5251		+4.8638	-0.2769
135.7644	169.7056	+0.4863	-9.0988	-5.8117	+0.5210	+5.7638	-4.7864
			+10.4758	+0.1101		+6.7638	-1.0297
235.3125	194.1406	+0.4782	-12.2593	-8.2178	+0.5339	+7.7925	-6.8453
			+13.6548	-0.3786		+8.7925	-1.8698
320.2864	400.3580	+0.4745	-14.4405	-9.8877	+0.5398	+9.1912	-8.2767
			+15.8446	-0.7295		+10.1912	-2.4620
509.1167	636.3959	+0.4699	-18.4103	-12.9377	+0.5472	+11.7353	-10.8938
			+19.8249	-1.3843		+12.7353	-3.5526

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear I_{sm}	Axial I_{am}	Circum. I_{cm}	Shear x_1/d	Axial x_2/d	Circum. x_3/d
6.7882	8.4853	-0.0275	+0.7442	+1.0524	0.4891	0.3233	0.5497
			+0.2558	+0.9844			
16.9706	21.2132	-0.0292	+0.9109	+1.0882	0.3151	0.2103	0.3534
			+0.0891	+0.9737			
46.4815	58.1018	-0.0333	+1.2739	+1.1661	0.1883	0.1250	0.2115
			-0.2739	+0.9505			
67.8822	84.8528	-0.0348	+1.4774	+1.2098	0.1549	0.1024	0.1740
			-0.4774	+0.9375			
135.7644	169.7056	-0.0372	+1.9803	+1.3178	0.1083	0.0712	0.1218
			-0.9803	+0.9054			
235.3125	294.1406	-0.0389	+2.5343	+1.4367	0.0816	0.0535	0.0919
			-1.5343	+0.8700			
320.2864	400.3580	-0.0397	+2.9217	+1.5199	0.0697	0.0456	0.0785
			-1.9217	+0.8452			
509.1167	636.3959	-0.0407	+3.6324	+1.6725	0.0550	0.0359	0.0620
			-2.6324	+0.7908			

TABLE 20 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, α = 60 DEG, T/t = 1.0

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sj}	I_{aj}	I_{cj}	I_{sj}	I_{aj}	I_{cj}
8.4853	8.4853	+0.4491	-0.8207	-0.0888	+0.4008	+0.8021	-0.0884
			+1.7835	+0.6820		+1.8021	+0.6928
21.2132	21.2132	+0.4326	-1.8758	-0.7923	+0.4339	+1.8763	-0.8259
			+2.8768	+0.5666		+2.8763	+0.5997
58.1018	58.1018	+0.4142	-3.7437	-2.2353	+0.4706	+3.7655	-2.2780
			+4.7872	+0.2515		+4.7655	+0.2813
84.8528	84.8528	+0.4087	-4.7130	-3.0202	+0.4816	+4.7410	-3.0625
			+5.7691	+0.0565		+5.7410	+0.0822
169.7056	169.7056	+0.4007	-7.0167	-4.9245	+0.4977	+7.0541	-4.9637
			+8.0914	-0.4480		+8.0541	-0.4312
294.1406	294.1406	+0.3959	-9.4887	-6.9933	+0.5073	+9.5293	-7.0292
			+10.5721	-1.0220		+10.5293	-1.0116
400.3580	400.3580	+0.3937	-11.9244	-8.4299	+0.5117	+11.2379	-8.4641
			+12.2833	-1.4283		+12.2379	-1.4214

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sm}	I_{am}	I_{cm}	x_1/d	x_2/d	x_3/d
8.4853	8.4853	-0.0265	+0.7634	+1.0566	0.4475	0.2992	0.5017
			+0.2366	+0.9832			
21.2132	21.2132	-0.0269	+0.9230	+1.0908	0.2911	0.1972	0.3253
			+0.0770	+0.9730			
58.1018	58.1018	-0.0292	+1.2595	+1.1630	0.1759	0.1192	0.1965
			-0.2595	+0.9514			
84.8528	84.8528	-0.0301	+1.4453	+1.2030	0.1451	0.0982	0.1622
			-0.4453	+0.9395			
169.7056	169.7056	-0.0315	+1.9006	+1.3007	0.1020	0.0688	0.1141
			-0.9006	+0.9105			
294.1406	294.1406	-0.0325	+2.3994	+1.4078	0.0771	0.0519	0.0863
			-1.3994	+0.8786			
400.3580	400.3580	-0.0329	+2.7471	+1.4824	0.0659	0.0443	0.0738
			-1.7471	+0.8564			

TABLE 21* STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, α = 60 DEG, T/t = 1.2

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sj}	I_{aj}	I_{cj}	I_{sj}	I_{aj}	I_{cj}
10.1823	8.4853	+0.3729	-0.6302	-0.0595	+0.4040	+0.9875	-0.1982
			+1.4357	+0.5152		+1.9875	+0.7033
25.4558	21.2132	+0.3626	-1.4475	-0.6675	+0.4288	+2.1814	-0.9497
			+2.2768	+0.3256		+3.1814	+0.6297
69.7222	58.1018	+0.3504	-2.9082	-1.9365	+0.4580	+4.3051	+2.4300
			+3.7656	-0.0833		+5.3051	+0.4974
101.8233	84.8528	+0.3468	-3.6732	-2.6299	+0.4667	+5.4067	+3.2259
			+4.5347	-0.3170		+6.4067	+0.3181
203.6467	169.7056	+0.3416	-5.4808	-4.3149	+0.4793	+8.0243	+5.1508
			+6.3586	-0.9089		+9.0243	-0.0362
254.5583	212.1320	+0.3402	-6.2071	-4.9974	+0.4825	+9.0725	+5.5289
			+7.0881	-1.1527		+10.0725	0.1854
352.9687	294.1406	+0.3385	-7.4258	-6.1473	+0.4866	+10.8303	+7.2391
			+8.3107	-1.5671		+11.8303	-0.4409
480.4296	400.3580	+0.3371	-8.7698	-7.4202	+0.4900	+12.7679	+8.6886
			+9.6579	-2.0297		+13.7679	-0.7281

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sm}	I_{am}	I_{cm}	x_1/d	x_2/d	x_3/d
10.1823	8.4853	-0.0258	+0.7811	+1.0604	0.4146	0.2792	0.4640
			+0.2189	+0.9820			
25.4558	21.2132	-0.0255	+0.9398	+1.0944	0.2706	0.1850	0.3019
			+0.0602	+0.9719			
69.7222	58.1018	-0.0269	+1.2673	+1.1647	0.1645	0.1128	0.1834
			-0.2673	+0.9510			
101.82238	84.8528	-0.0275	+1.4463	+1.2032	0.1360	0.0932	0.1517
			-0.4463	+0.9395			
203.6467	169.7056	-0.0284	+1.8827	+1.2968	0.0959	0.0656	0.1070
			-0.8827	+0.9116			
254.5583	212.1320	-0.0286	+2.0598	+1.3349	0.0857	0.0586	0.0956
			-1.0598	+0.9003			
352.9687	294.1406	-0.0290	+2.3587	+1.3990	0.0727	0.0497	0.0811
			-1.3587	+0.8812			
480.4296	400.3580	-0.0293	+2.6901	+1.4702	0.0622	0.0425	0.0694
			-1.6901	+0.8600			

TABLE 22 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH A CONICAL HEAD, α = +60 DEG, T/t = 1.6

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension: - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sj}	I_{aj}	I_{cj}	I_{sj}	I_{aj}	I_{cj}
13.5764	8.4853	+0.2809	-0.3878	-0.0296	+0.4001	-1.2624	-0.3448
			+0.9891	+0.2778		+2.2624	+0.7126
33.9411	21.2132	+0.2773	-0.8994	-0.5232	+0.4117	-2.5828	+1.1339
			+1.5091	-0.0120		+3.5828	+0.7158
92.9630	58.1018	+0.2717	-1.8236	-1.5769	+0.4297	-4.9652	+2.6500
			+2.4461	-0.5572		+5.9652	+0.6291
135.7644	84.8528	+0.2700	-2.3069	-2.1559	+0.4350	-6.2076	+3.4607
			+2.9334	-0.8533		+7.2076	+0.5638
271.5289	169.7056	+0.2676	-3.4604	-3.5658	+0.4427	-9.1675	+5.4161
			+4.0924	-1.5748		+10.1675	+0.3844
339.4111	212.1320	+0.2670	-3.9224	-4.1199	+0.4446	-10.3544	+6.2054
			+4.5567	-1.8708		+11.3544	+0.3072
470.6249	294.1406	+0.2662	-4.7001	-5.1009	+0.4471	-12.3452	+7.5334
			+5.3553	-2.3683		+13.3452	+0.1737

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sm}	I_{am}	I_{cm}	x_1/d	x_2/d	x_3/d
13.5764	8.4853	-0.0247	+0.8105	+1.0667	0.3647	0.2474	0.4074
			+0.1895	+0.9802			
33.9411	21.2132	-0.0237	+0.9709	+1.1011	0.2382	0.1641	0.2653
			+0.0291	+0.9699			
92.9630	58.1018	-0.0241	+1.2937	+1.1704	0.1456	0.1008	0.1620
			-0.2937	+0.9493			
135.7644	84.8528	-0.0244	+1.4684	+1.2079	0.1207	0.0836	0.1342
			-0.4684	+0.9381			
271.5289	169.7056	-0.0247	+1.8917	+1.2988	0.0854	0.0592	0.0950
			-0.8917	+0.9110			
339.4111	212.1320	-0.0249	+2.0631	+1.3356	0.0764	0.0529	0.0849
			-1.0631	+0.9001			
470.6249	294.1406	-0.0250	+2.3517	+1.3975	0.0648	0.0449	0.0721
			-1.3517	+0.8816			

TABLE 23 STRESS INDEXES IN A CYLINDRICAL PRESSURE VESSEL WITH CONICAL HEAD, α = 60 DEG, T/t = 2.0

(Upper values refer to stress at external surface; lower values refer to stress at internal surface. + = tension; - = compression.)

		Cone at Junction			Cylinder at Junction		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sj}	I_{aj}	I_{cj}	I_{sj}	I_{aj}	I_{cj}
16.9706	8.4853	+0.2280	-0.2460	-0.0182	+0.3871	+1.4310	-0.4481
			+0.7195	+0.1226		+2.4310	+0.7105
42.4264	21.2132	+0.2279	-0.5820	-0.4469	+0.3875	+2.7752	-1.2376
			+1.0557	+0.2210		+3.7752	+0.7276
116.2037	58.1018	+0.2256	-1.1909	-1.3726	+0.3965	+5.2213	-2.7456
			+1.6698	+0.8421		+6.2213	+0.6872
169.7055	84.8528	+0.2249	-1.5099	-1.8826	+0.3994	+6.5008	3.5499
			+1.9905	+1.1726		+7.5008	+0.6506
339.4111	169.7056	+0.2239	-2.2718	-3.1261	+0.4035	+9.5532	-5.4870
			+2.7548	+1.9712		+10.5532	+0.6799
424.2639	212.1320	+0.2236	-2.5778	-3.6301	+0.4045	+10.7785	-6.2682
			+3.0614	+2.2943		+11.7785	-0.4986

		Maxima in Cylinder			Location of Maxima in Cylinder		
d/t	d/T	Shear	Axial	Circum.	Shear	Axial	Circum.
		I_{sm}	I_{am}	I_{cm}	x_1/d	x_2/d	x_3/d
16.9706	8.4853	-0.0236	+0.8315	+1.0712	0.3280	0.2232	0.3663
			+0.1685	+0.9788			
42.4264	21.2132	-0.0221	+0.9911	+1.1054	0.2140	0.1477	0.2382
			+0.0089	+0.9686			
116.2037	58.1018	-0.0220	+1.3082	+1.1735	0.1311	0.0910	0.1457
			-0.3082	+0.9483			
169.7055	84.8528	-0.0220	+1.4790	+1.2102	0.1088	0.0756	0.1209
			-0.4790	+0.9374			
339.4111	169.7056	-0.0221	+1.7921	+1.2989	0.0771	0.0536	0.0856
			-0.7921	+0.9110			
424.2639	212.1320	-0.0222	+2.0591	+1.3347	0.0690	0.0480	0.0766
			-1.0591	+0.9003			

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* Correction to Table 21. For d/t of 254.5583, under Cylinder at Junction, *I_{cj}* of 5.5289 should be 5.9289.

$$\delta = \frac{1}{2\beta^2 D}\left(\beta M_o + Q_o\right) - \frac{pd^2}{8Et}\left(2-\mu\right)$$

$$\delta^{'} = -\frac{1}{2\beta^2 D}\left(2\beta M_o + Q_o\right)$$

These equations may be rearranged to

$$\frac{ET\delta}{pd^2} = \left[-\frac{m^2}{2}\frac{d}{t}\frac{M_o}{pd^2} - \frac{m}{2}\sqrt{\frac{d}{t}}\frac{Q_o}{pd} + \left(\frac{2-\mu}{8}\right)\right]\frac{T}{t}$$

$$\frac{ET\delta^{'}}{m^2 pd^2} = -\left[m\sqrt{\frac{d}{t}}\frac{M_o}{pd^2} + \frac{1}{2}\frac{Q_o}{pd}\right]\left(\frac{T}{t}\right)^2$$

Comparing with Equations [1] and [2]

$$a_4 = \frac{m^2}{2}\frac{d}{t}\frac{T}{t} \qquad b_4 = -m\sqrt{\frac{d}{t}}\left(\frac{T}{t}\right)^2$$

$$a_5 = \frac{m}{2}\frac{T}{t}\sqrt{\frac{d}{t}} \qquad b_5 = -\frac{1}{2}\left(\frac{T}{t}\right)^2$$

$$a_6 = \left(\frac{2-\mu}{8}\right)\frac{T}{t} \qquad b_6 = 0$$

INFLUENCE NUMBERS FOR THE CONICAL HEAD

Influence numbers for the conical head are determined by a procedure identical to that just illustrated. The final results are

$$a_1 = \frac{b_2 \xi_o{}^2 \tan\alpha \sin\alpha}{2}$$

$$a_2 = \frac{\frac{\xi_0{}^2 B}{4} - \mu^2 G}{C + 2\mu G}\sin\alpha$$

$$a_3 = \frac{2-\mu}{8}\sec\alpha - \frac{a_2}{4}\tan\alpha + \frac{3b_2(1+\mu)}{\xi_0{}^2}\sec\alpha$$

$$b_1 = \frac{2G}{C + 2\mu G}$$

$$b_2 = \frac{A/2}{C + 2\mu G}$$

$$b_3 = \frac{6(1+\mu)}{\xi_o{}^4} b_1 \csc^2\alpha - \frac{b_2\tan\alpha}{4} - \frac{3}{2}\frac{\sec\alpha\csc\alpha}{\xi_o{}^2}$$

where

$$A = \xi_o(ber_2{}'\xi_o bei\xi_o - bei_2{}'\xi_o ber_2\xi_o)$$

$$B = \left(ber_2{}'\xi_o\right)^2 + \left(bei_2{}'\xi_o\right)^2$$

$$C = \xi_o\left(ber_2\xi_o ber_2{}'\xi_o + bei_2\xi_o bei_2{}'\xi_o\right)$$

$$G = \left(ber_2\xi_o\right)^2 + \left(bei_2\xi_o\right)^2$$

The formulas for the *ber* and *bei* functions are given by N. W. MacLachlan.' Asymptotic expressions for these functions and their derivatives are accurate to four or more significant figures whenever ξ_o exceeds 8.

The calculations are based on the relations

$$ber_o\xi_o = M_o\cos\theta_o \qquad bei_o\xi_o = M_o\cos\theta_o$$

$$ber_1\xi_o = M_1\cos\theta_1 \qquad bei_1\xi_o = M_1\sin\theta_1$$

The asymptotic formulas for M_o,.M_1, θ_o,.θ_1, are given by MacLachlan.[9] The functions $ber_2\ \xi_o$ and $bei_2\ \xi_o$ and their derivatives can be found from the same reference.[10]

The exponential factor common to the asymptotic expressions for M_o and M_1 can be omitted in the calculations because the quantities A, B, C, and G appear only as ratios in the definitions of a_2, b_1, and b_2.

MAXIMUM STRESSES IN CYLINDER

The ordinary theory of maxima and minima can be applied to determine the maximum stress index (of any type) in the cylinder away from the junction. The expressions of Table I imply that the location and magnitude of the shear - stress index and axial - stress index are determined by the value of the shear - stress index at the junction and by the ratio $\lambda = \beta M_o/Q_o$. The remark also applies to the circumferential - stress index. This index is omitted from Table I because its complete specification has many possible forms depending upon the sign and magnitude of both Q_o and M_o.[11]

MAXIMUM STRESSES IN CONICAL HEAD

Analogous to the maximum stresses in the cylinder, there are maximum stresses in the conical head away from the junction. These could be determined by further computations. However, in contrast to the cylinder, it is not possible to establish simple expressions for the three stress indexes since it is quite tedious to compute the values of ξ which establish the location of these indexes.

Discussion

H. C. BOARDMAN [12] This paper is just what it purports to be,namely, an analysis based on the theory of elasticity, giving the reader a clear mental picture of the elastic behavior of the cylinder - cone junction regions of originally ideal, isotropic, stressfree, and seamless pressure vessels, each consisting of a perfect cylindrical shell and two perfect conical heads directly joined thereto. This analysis indicates that for many—perhaps for most—commercial pressure vessels, each made of one cylinder and two conical heads, the elastic action covered by the paper occurs only during the first part of the ordinary hydrostatic test to 11/2 times the design pressure, and is followed in the later stages of the test by some plastic deformation which gives the cylinder - head region a new form and sets up a system of residual stresses such that the subsequent behavior of the vessel in service, although wholly elastic, is not calculable from the theory of elasticity because the vessel shape and the pattern and intensity of the locked - up stresses are unknown.

For example, the sample problem presented by the authors records computed stresses varying from 107,000 psi to 163,000 psi depending upon the *T/t* ratio, and explains them by stating that "the magnitude of these stresses indicates, of course, that some yielding and stress relief has occurred in the neighborhood of the junction between the body and head." From one point of view the term "stress relief" is a misnomer in this case since it implies that the calculated stress intensities were reached and then reduced; actually, plastic deformation prevents stresses of the calculated intensities and leaves residual stresses when the test pressure is reduced to zero.

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7 The sign of quantities δ, M_o, Q_o are changed, β is replaced by $m/\sqrt{dt}, D\ \ by\ \ Et^3/m^4, h\ \ by\ \ t, a\ \ by\ \ d/2$

8 "Bessel Functions for Engineers," by N. W. MacLachlan, Oxford University Press, London, England, 1934.

[9] See footnote 8 equations [59] to [62], p. 133.

[10] See footnote 8, equations [159] and [l60], p. 168, equations [15] to [19], p. 158.

[11] Details of the results and procedure can he obtained from the second author.[3]

[12] Director of Research, Chicago Bridge and Iron Company, Chicago, Ill. Mem. ASME.

The sample problem and the foregoing quoted comment thereon constitute an acknowledgment by the authors that the theory of elasticity is unable to solve fully the basic problem of the cylinder-cone junction. If, perforce, wholly fictitious and really impossible calculated stresses must be involved as one link in a chain leading to the solution of this problem, then, eventually, they must in some usable way be related to the actual change of shape and residual-stress system and evaluated in terms of service performance and safety.

The writer has a growing conviction that the Pressure Vessel Research Committee (PVRC) of the Welding Research Council should and must assume the task of and responsibility for interpreting such papers as the authors' in terms which will throw light on the major job of deciding how to arrive at safe but e economical designs, taking into account the plastic deformation occurring during the hydrostatic test and the residual-stress system established thereby. This, the writer believes, is what the financial sponsors of the PVRC want, not only with reference to conical heads, without knuckles and compression rings, with compression rings but without knuckles, and with knuckles without compression rings; but also with reference to the shallower torispherical and ellipsoidal heads.

The authors have done a splendid analytical job and their paper in conjunction with others of its kind and the results of the work of the PVRC Design Division at Purdue University, provides a foundation upon which, by an exercise of combined experience, experiment, analysis, and judgment, the PVRC must, if it attains its avowed objective, reach specific workable conclusions which, in effect, will confirm the ASME Code rules of design as they are, or recommend revisions thereof, or advocate entirely new rules.

The authors deserve commendation for preparing a very thought-provoking paper. Following are some of the questions which it raises in the writer's mind:

1 Are not conclusions 1 and 2 self-evident? It appears axiomatic that a cylinder under internal gas pressure expands; that the base of a cone suspended on a vertical canvas cylinder and under internal gas pressure contracts since the canvas cannot resist the components perpendicular to the axis of the along-the-element stresses; and that the tendency of the cylinder to expand and of the cone base to contract must cause discontinuities at their junction.

2 Are localized working stresses of high intensity just as dangerous to the vessel as general stresses of the same intensity would be? Put in another way, would a streamlined vessel with a general working stress intensity equal to the localized working stress intensity of a conventional vessel (such as that of the sample problem) be just as safe as the latter.

If the answer is “yes,” then industry should simultaneously streamline its pressure vessels and raise the design stress, thus promoting both safety and economy in materials.

3 What is the optimum test pressure in terms of the working pressure and nominal working stress? Otherwise expressed, what test pressure will establish the optimum residual stresses and how can it be determined?

4 Should there be any design limit, short of that which will produce buckling, on the ring compressive stress at a cylinder - cone junction? Put in another way, are all of the stress factors of Table 3 of equal significance? Should they all he held to the same numerical value?

5 The authors state, “Where it is not possible to have the thickness the same in both cone and cylinder, it is more desirable to make the cone thicker.” Under what conditions would it be “not possible” to have the cone and cylinder the same thickness? Do the words "not possible" imply an excessive ring stress in the cone when the apex angle is large?

6 How does a weld at the cylinder-cone junction affect the strains and stresses in the junction regions?

Such queries haunt and hound the thoughtful reader of this paper; perhaps its greatest value lies in this fact.

AUTHORS’ CLOSURE

The authors wish to thank Mr. Boardman for his careful and thought-provoking review of the paper. They wish to offer a number of comments in answer to his points.

Mr. Boardman's first paragraph indicates that plastic flow will probably occur at some points of a pressure vessel during the hydrostatic test but that the subsequent behavior in service will be wholly elastic. Since the volume of material involved in plastic flow may be extremely small, it seems entirely possible that some plastic deformation may occur each time the vessel is loaded or unloaded due to elastic stress in the other parts of the vessel. Such a wringing action could eventually result in a crack. Thus the high stresses indicated by elastic analysis are valuable since they point out locations where trouble may be expected.

In locations where it is possible for plastic flow to occur, the maximum stress will be limited to the flow stress and the high peaks indicated by elastic analysis will be redistributed by increasing the stress to the flow value at nearby points. The authors agree that the term “stress relief” for this effect is a misnomer and “stress redistribution” might be preferable.

Continuing his discussion, Mr. Boardman points out that because of the plastic flow which intervenes at points of stress concentration, the results of elastic analysis must be “evaluated” or “interpreted” before being applied to practical vessels. The authors believe that this is an oversimplified view of the situation. Having made an elastic analysis, as in this paper, it then becomes necessary to analyze those small areas of the vessel where plastic flow is indicated. To compute the plastic stresses and strains in such areas is another problem of considerable difficulty in itself. Although these phases of the over-all design problem are only beginning to be investigated by rational methods it will be necessary to have such solutions before conclusions on the safety of practical vessels can be drawn.

Mr. Boardman concluded by asking six questions. The authors' comments on these are as follows:

1 The "obvious" conclusions 1 and 2 were drawn to call attention to a basic defect in any design having sharp corners. It has been shown that severe stress concentrations are set up at such points. In many practical cases plastic flow permits a vessel to resist the loads without damage. However, there are also circumstances where plastic flow is impossible so that such a design is unsafe. Two examples may be mentioned: (*a*) Ordinary structural steel at low temperatures; (*b*) low-alloy steels which have a high yield point obtained at the expense of reduced ductility.

2 The authors believe that a design with stresses of high intensity throughout the vessel would be much more likely to fail than one where high stresses are confided to small areas. The following comments may be made: (*a*) When the high stress is localized, the surrounding elastic region exerts restraint on the region of high stress and makes the vessel stronger than if the high stress were general. (*b*) If a single local crack developed in either case, the localized stress is safer because the surrounding elastic region has, at least potentially, strain energy available up to rupture to absorb the energy released by tile crack. (*c*) A general high stress is likely to produce cracks at some weak point of a structure. On grounds of probability a localized high stress may not occur at a weak point.

3 The answer to this question requires analysis of the plastic regions as discussed above. Both the elastic and plastic analyses are essential for a complete solution to these problems. The elastic analysis is needed to define the areas where plastic flow may occur. If residual stresses are set up by plastic flow, the elastic analysis is still needed to compute the total stress at a point during cycles of loading and unloading. It seems obvious that approximate elastic theories would be of little use since precise values of stress at all points must be known if a combined elastic and plastic case is to be correctly analyzed.

4 The authors agree that, in general, the significance of positive and negative stresses will be different. The importance of ring compressive stresses was emphasized in the recent paper by Messrs. Wilkin and Wetterstrom.[13]

It is mainly in recent years that rational methods have been applied in studying the flow of metals under biaxial or triaxial resses Considerable progress has been made on problems of forming drawing, rolling, etc., but much less attention has been given to the conditions under which failure of the material will occur.. This is certainly an area where future research work should make a concentrated attack.

5 The authors had in mind the occasional assembly of small vessels from salvaged components, as might be necessary for emergency repairs of experimental work. It is also conceivable that it might be necessary to use a thicker head to provide sufficient metal for reinforcement around openings which might be necessary after the vessel is erected and which might be difficult to otherwise reinforce.

6 A large amount of current research is devoted to welding. Several years will probably elapse before firm conclusions can be drawn regarding the behavior of welds under structural loads.

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[13] "Lateral Buckling of Circular Stiffening Rings in Compression," by L A. Wilkin and Edwin Wetterstrom. Presented at the Annual Meeting of THE. AMERICAN SOCIETY OF MECHANICAL ENGINEERS, Atlantic City. N. J., Nov 25 - 30, 1951.